P1

# Review of EE 205

26 Eylül 2020 Cumartesi 17:16

Electric potential (voltage): Work done by 1C charges $:= \frac{E}{q}$

E = Electric potential energy. (Joules)

q = charge. (Coulombs)

Current $= \frac{q}{t}$ (A) = I

q = charge.

t = time. (sec.)

Power $= \frac{E}{t}$ (watts). $= \underbrace{\frac{E}{q}}_{V} \cdot \underbrace{\frac{q}{t}}_{I} = V.I$ (W).

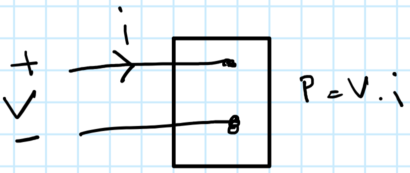
## Ideal Basic Circuit Elements:

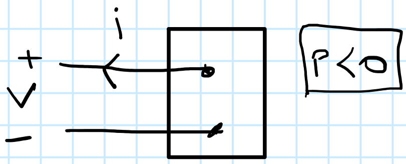
It is an electrical component with the following properties:

1-) Two terminals.

2-) Can be described as voltage or current.

3-) Can not be subdivided into other elements.





If the current is going out of the circuit element, this refers to the existence of an energy source (generator).

If the power consumption by the element is positive, (P > 0), the power is being delivered to the circuit inside the box.

If the power is negative, P<0, the power is being extracted from the circuit inside the box.

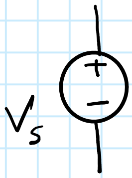
## Chapter 2 : Circuit Elements :

There are two circuit elements: voltage and current sources.

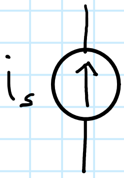
- Ideal voltage source: Provides a constant voltage across its terminals regardless of the current.
- Ideal current source: Similar to the voltage source. It provides current across its terminals regardless of the voltage.

If circuit elements do not depend on any other parameter, they are called "independent sources".

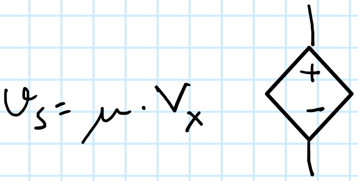
Circuit symbols:



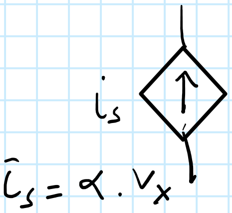
Ideal Independent voltage source.



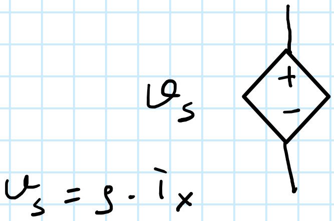
Ideal indep. current source.



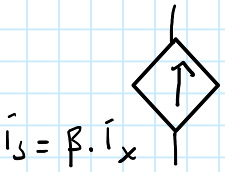
Ideal dependent voltage controlled voltage source.



Ideal dependent voltage controlled current source.



Ideal dependent current controlled voltage source.



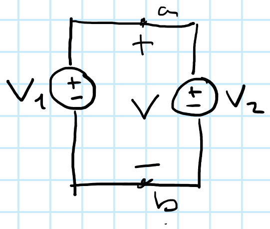
Ideal dependent current controlled current source.

# P3

Thursday, March 2, 2023 9:55 AM

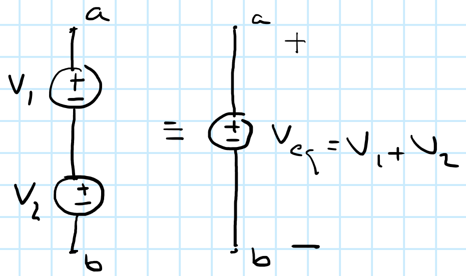
## Constraints for the Connection of Circuit Elements:

- Two voltage sources:

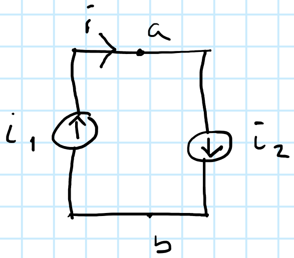


$\Rightarrow \boxed{V_1 = V_2 = V}$

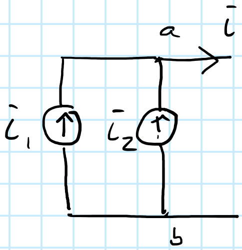
Also, the polarities must be the same.



- Two current sources:



$\Rightarrow \boxed{i_1 = i_2 = i}$

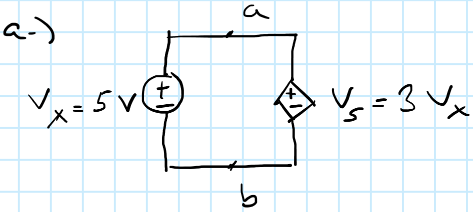


$\Rightarrow i = i_1 + i_2$

**Ex:**

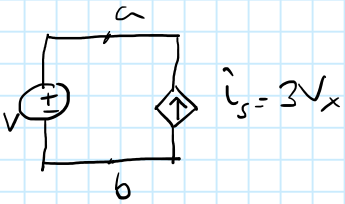
Determine which connections are valid.

a.)



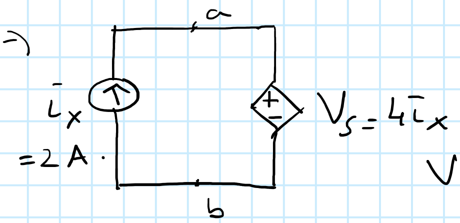
Not valid.

b.)



$\Rightarrow$ Ideal voltage source supplies the same voltage regardless of the current, and visa versa. Thus, this is valid.

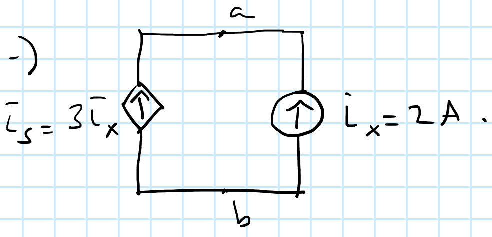
c.)



Valid.

$\Rightarrow$ Because of the same reason in part b.

d.)



$\Rightarrow$ Not valid. $i_x \neq i_s$

# P4

Thursday, March 2, 2023 10:06 AM

Two important concepts:

- Active element: A device capable of generating electrical energy. $(p<0)$
- Passive element: A device that can not generate electrical energy. $(p>0)$. Examples: Resistors, inductors, capacitors. (lumped elements).

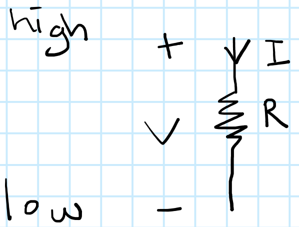
Resistance:

is the capacity of materials to resist (impede, oppose) the current flow.

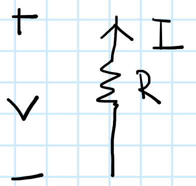
Ohm's law : $\boxed{V = I \cdot R}$

Reciprocal of resistance is called "conductance" with a symbol "G".

$\Rightarrow$ $G = \frac{1}{R}$ (Siemens or ℧) (S) ↳ "mho"  R ($\Omega$ = ohm)



$V = IR$



$V = -IR$

$\boxed{G = \frac{1}{R} \cdot (S)}$

$\boxed{P = VI = I^2R\ (W).}$

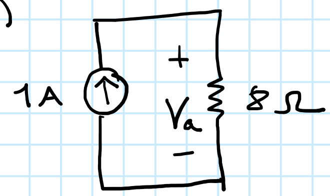
Also, $\boxed{P = \frac{V^2}{R}\ (W)}$

$\boxed{I = VG}$ $\boxed{P = V^2 \cdot G}$

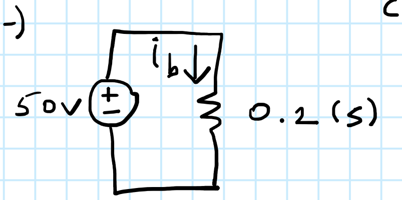
Ex:

Calculate the values of $v$ and $i$, and determine the power dissipated in each resistor.

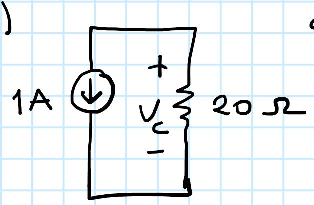
a-)



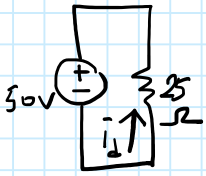
b-)



c-)



d-)



Ans:

a-) $V_a = i \cdot R = (1A)(8\Omega) = 8\,V.$ , $P_{8\Omega} = \frac{V^2}{R} = \frac{8^2}{8} = 8$ Watts.

b-) $i_b = \frac{V}{R} = V \cdot G = (50V)(0.2) = 10\ A.$ , $P_{0.2} = V^2G = 500\ W.$

# P5

Thursday, March 2, 2023 10:37 AM

c-) $V_c = i \cdot R = \underbrace{-(1A)}_{i}(20\,\Omega) = -20V$, $P_{20\Omega} = \frac{V^2}{R} = \frac{(-20)^2}{20} = 20W.$

d-) $V = -i \cdot R$

$\Rightarrow 50 = -i_d(25)$

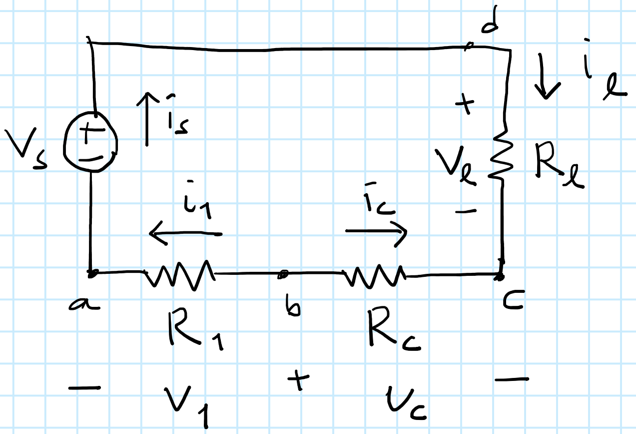
$\Rightarrow i_d = -2A$

, $P = \frac{V^2}{R} = (-50)^2/25 = 100\,W.$

– sign indicates that the original selection of the current direction is really the opposite.

## Kirchoff's Laws:

Suppose we have the following circuit,



$V_1 = i_1 R_1$

$V_c = i_c R_c$

$V_\ell = i_\ell R_\ell$

We consider leaving currents positive, and entering currents negative.

## Kirchoff's Current Law (KCL):

The algebraic sum of the currents at any point in a circuit is zero.

At point a:

$i_s - i_1 = 0$

At point b:

$i_1 + i_c = 0$

At point c:

$-i_c - i_\ell = 0$

At point d:

$i_\ell - i_s = 0$

Note that initial assumption of currents and voltages are not important. At the end of the solution, we get results which are + or –, indicating the real directions.

# P6

Thursday, March 2, 2023 10:52 AM

## Kirchoff's Voltage Law (KVL):

The algebraic sum of all the voltages around any closed path in a circuit is zero.

Closed path: dcbad (we follow clockwise direction.)

$v_e - v_c + v_1 - v_s = 0$ (KVL equation.)

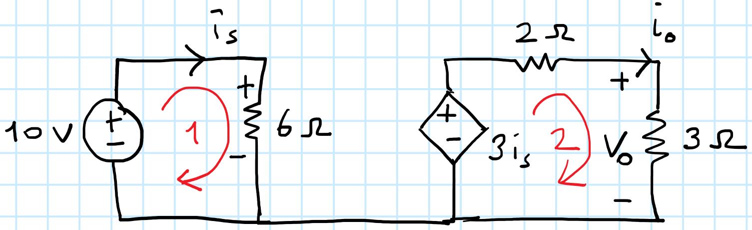
(+ → − is always positive.)

Closed path: dabcd (ccw)

$V_s - V_1 + V_c - V_e = 0$ (The same equation as before.)

Ex:

Given the circuit below.



a-) Find $V_o$ from KVL and Ohm's law.

b-) Show that $P_{total}\Big|_{dissipated} = P_{total}\Big|_{developed}$ (Conservation of energy.)

Ans:

a-) KVL in loop 1:

$-10V + 6 i_s = 0$

$\Rightarrow i_s = \frac{10}{6} = \frac{5}{3} A.$

Also, KVL for loop 2:

$-3 i_s + 2 i_o + V_o = 0$

$-5 + 2 i_o + V_o = 0$

Ohm's law:

$V_o = 3 i_o$

Thus,

$-5 + 2 i_o + 3 i_o = 0$

$5 i_o = 5$

$\Rightarrow i_o = 1 A.$

Then, $V_o = 3 i_o = 3V.$

b-) Power for indep. voltage source:

$P = vi = (-10v) . i_s = 10.(\frac{5}{3}) = -16.7W$

Also,

$P_{6\Omega} = vi = (10v) i_s = \boxed{16.7W}$

$P_{2\Omega} = i^2 R = i_o^2 (2) = (1)^2 (2) = \boxed{2W.}$

$P_{3\Omega} = i^2 R = i_o^2 (3) = (1)^2 (3) = \boxed{3W.}$

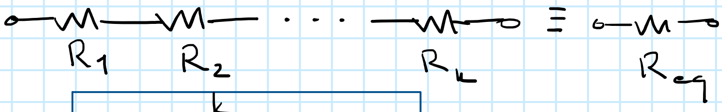
# HW2

## P7

Thursday, March 9, 2023 9:32 AM

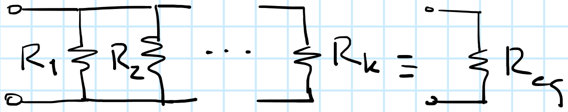
## Resistive Circuits:

### Resistors in Series:



$$\boxed{R_{eq} = \sum_{i=1}^{k} R_i \quad (\Omega)}$$

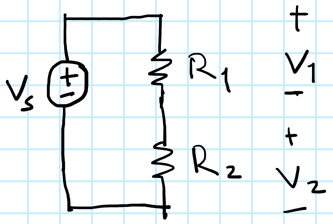
### Resistors in Parallel:



$$\boxed{R_{eq} = \frac{1}{\sum_{i=1}^{k} \left(\frac{1}{R_i}\right)} \quad (\Omega)}$$

## Voltage Divider:

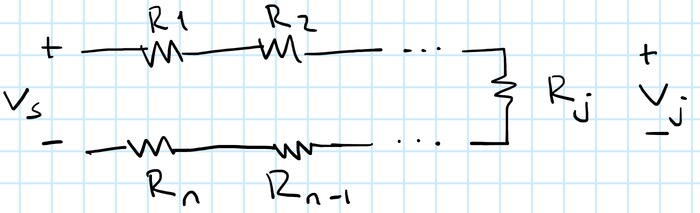
- Short method to find voltages when several resistors are connected in series.



$$\boxed{U_1 = U_s \cdot \frac{R_1}{R_1 + R_2}}$$

$$\boxed{V_2 = V_s \cdot \frac{R_2}{R_1 + R_2}}$$

In case of n resistors connected in series:

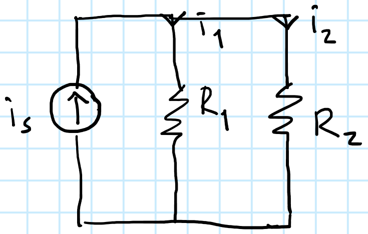


$$\boxed{V_j = U_s \cdot \frac{R_j}{R_{eq}}},$$

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

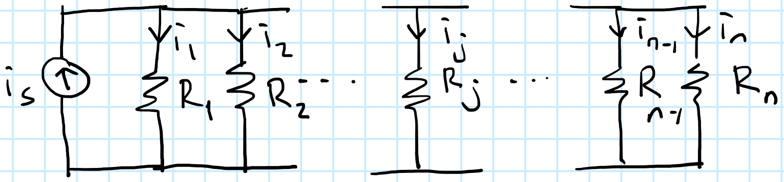
# Current Divider:

It is a rule for finding the current passing through the resistors which are connected in parallel.



$$\boxed{i_1 = i_s \cdot \frac{R_2}{R_1+R_2}} \quad , \quad \boxed{i_2 = i_s \cdot \frac{R_1}{R_1+R_2}}$$

For general current division, we have:

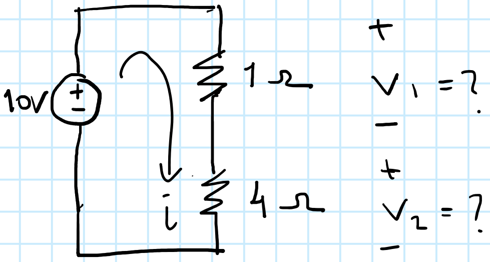


$$\boxed{i_j = i_s \cdot \frac{R_{eq}}{R_j}}$$

where

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

Ex:



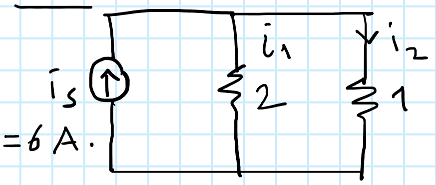
Ans:

By using the voltage division rule:

$$V_1 = 10 \cdot \frac{1}{1+4} = \frac{10}{5} = 2V.$$

$$V_2 = 10 \cdot \frac{4}{1+4} = \overset{2}{\cancel{10}} \cdot \frac{4}{\cancel{5}} = 8V.$$

Ex:



Find $i_1$, $i_2$.

Ans:

By using the current division

$$i_1 = i_s \cdot \frac{(1\Omega)}{(2+1)\Omega} = (\overset{2}{\cancel{6}}A) \cdot \frac{1}{\cancel{3}} = 2A.$$

$$i_2 = i_s - i_1 = 6 - 2 = 4A$$

Also, $i_2 = i_s \cdot \frac{2}{2+1} = \overset{2}{\cancel{6}} \cdot \frac{2}{\cancel{3}} = 4A.$ as befre.

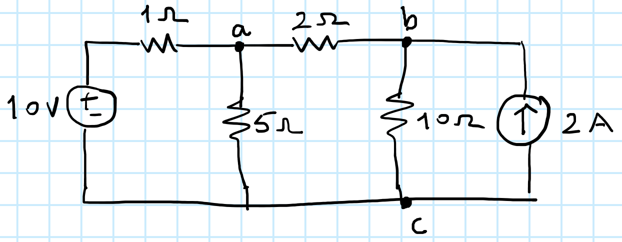
P9

Thursday, March 9, 2023 9:48 AM

# Cp. 4. Techniques of Circuit Analysis:

## 1-) Node-Voltage Method:

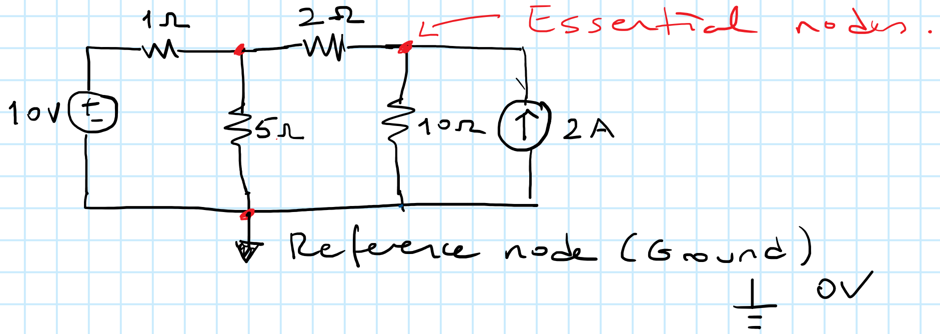
Suppose we have the following circuit:



**Step 1:** Simplify the circuit.

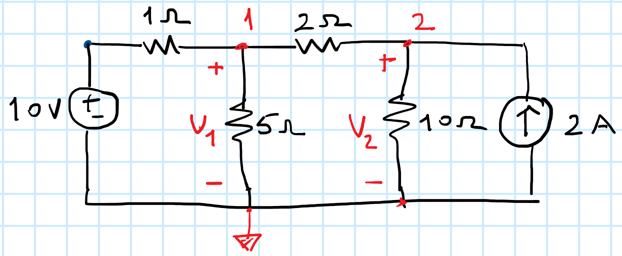
**Step 2:**

Select one of the nodes as a reference node.



**Step 3:**

Define node voltages w.r.t the reference node.



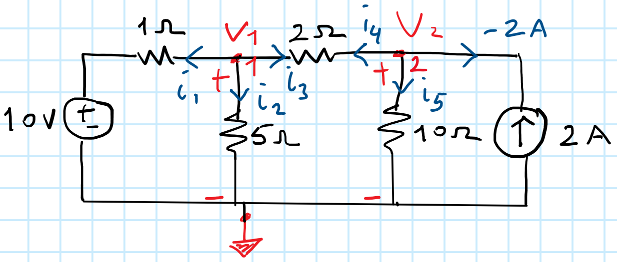
**Step 4:**

Generate node-voltage equations by:

# P10

Tuesday, March 14, 2023 12:47 PM

Writing the node equations (KCL) for each node employing the leaving current convention.



KCL at node 1 gives:

$$\underset{\downarrow}{i_1} + \underset{\searrow}{i_2} + \underset{\searrow}{i_3} = 0$$

$$\frac{V_1 - 10}{1\Omega} + \frac{V_1}{5\Omega} + \frac{V_1 - V_2}{2\Omega} = 0 \quad \text{———} (1)$$

KCL at node 2 gives: ———(2)

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2A = 0$$

Equations (1) and (2) can be solved simultaneously

Let us re-write (1)

$$\underset{(10)}{\frac{V_1}{1}} + \underset{(2)}{\frac{V_1}{5}} + \underset{(5)}{\frac{V_1}{2}} - \underset{(5)}{\frac{V_2}{2}} = 10$$

$$10V_1 + 2V_1 + 5V_1 - 5V_2 = 100$$

$$17V_1 - 5V_2 = 100 \quad \text{———} (3)$$

Re-write (2) as

$$\underset{(5)}{\frac{V_2}{2}} - \underset{(5)}{\frac{V_1}{2}} + \underset{(1)}{\frac{V_2}{10}} = 2$$

$$5V_2 - 5V_1 + V_2 = 20$$

$$-5V_1 + 6V_2 = 20 \quad \text{———} (4)$$

$$6 \,/\, 17V_1 - 5V_2 = 100 \Rightarrow 102V_1 - 30V_2 = 600 \Rightarrow 77V_1 = 700$$

$$5 \,/\, -5V_1 + 6V_2 = 20 \Rightarrow + \underline{-25V_1 + 30V_2 = 100}$$

$V_1 = 9.09$. and $V_2 = 10.9$

# P11

Thursday, March 9, 2023 10:28 AM

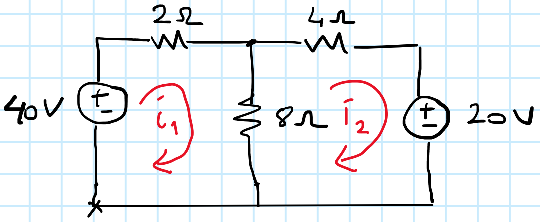
## 2-) Mesh-Current Method:

For example, consider the following circuit:

For mesh-current analysis, we write the KVL equations for each mesh, and solve the equations simultaneously.

**Ex:**

Use the mesh-current method to determine the power associated with each voltage source.



**Ans:**

KVL for mesh 1:

$$-40 + 2i_1 + 8(i_1 - i_2) = 0 \quad \text{———} (1)$$

KVL for mesh 2:

$$4i_2 + 20V + 8(i_2 - i_1) = 0 \quad \text{———} (2)$$

Solve (1) and (2) simultaneously as

$$3/\ 10i_1 - 8i_2 = 40 \quad \text{———} (4)$$

$$2/\ -8i_1 + 12i_2 = -20 \quad \text{———} (5)$$

$$30i_1 - 24i_2 = 120$$

$$+ \quad -16i_1 + 24i_2 = -40$$

$$14i_1 = 80$$

$$\Rightarrow i_1 = 5.7A. \quad \text{and}$$

$$57 - 8i_2 = 40$$

$$\Rightarrow 8i_2 = 57 - 40 = 17$$

$$\Rightarrow i_2 = \frac{17}{8} = 2.125A.$$

To find the power generated by 40V source:

$$P_{40V} = (-40V)(i_1) = (-40)(5.7) = -228W.$$

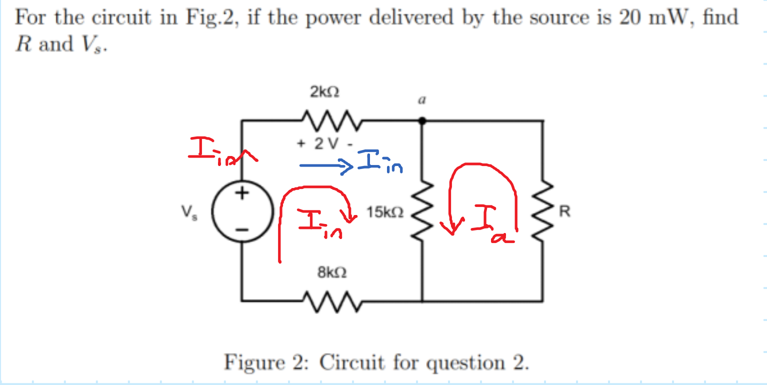
$$P_{20V} = (20V)(i_2) = (20)(2.125) = 42.5W.$$ (This may damage the source)

# P12

Thursday, March 9, 2023 11:14 AM

Solve the following questions. (The answers should be shown in the video recordings.)

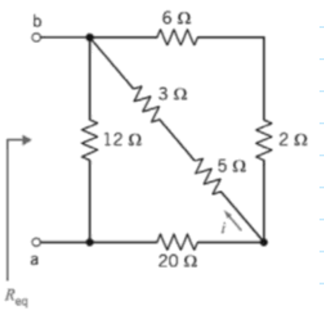
1-)



For the circuit in Fig.2, if the power delivered by the source is 20 mW, find $R$ and $V_s$.

Figure 2: Circuit for question 2.

2-)

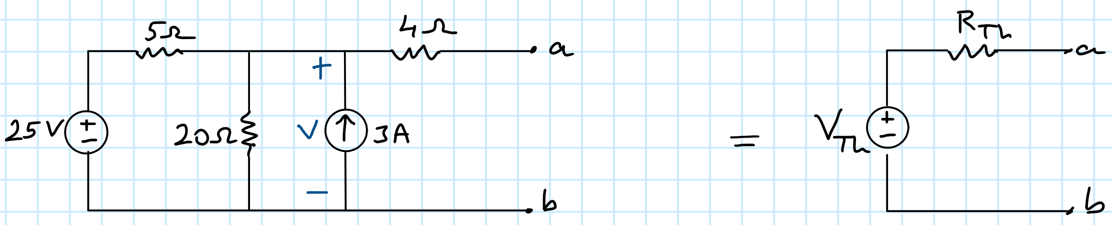


Find $i$ and $R_{eq}$ if $V_{ab} = 40V$

HW 3

## Thévenin and Norton Equivalents:

Consider the following circuit:



Thévenin Equavent Circuit.

- If we connect another circuit to the given circuit at points a and b, this other circuit sees no difference if it is connected to Thévenin equivalent circuit.

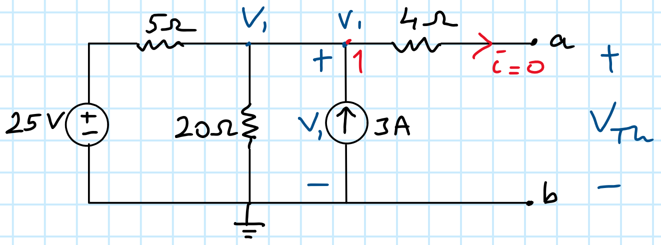
$V_{Th}$ = Open circuit voltage btw. the terminals a and b, and we short circuit a and b, and find the current btw a and b ($i_{sc}$).

$$\boxed{R_{Th} = \frac{V_{Th}}{i_{sc}}}$$

# P13

Thursday, March 16, 2023 9:44 AM

For the given circuit; using the node-voltage method:



The current passing through the 4Ω resistor is zero.

$V = i.R = 0$

$v_1 - v_{Th} = 0$

$\Rightarrow \boxed{V_1 = V_{Th}}$

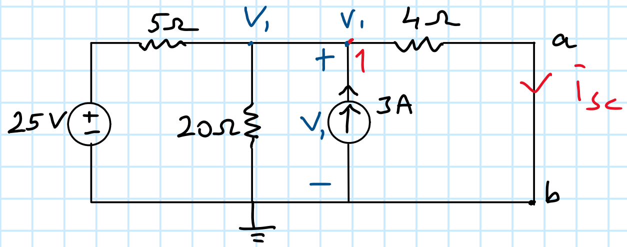
KCL at node 1:

$$\frac{V_1 - 25}{5} + \frac{V_1}{20} - 3 = 0$$

$\Rightarrow V_1 = 32V.$

$\boxed{V_{Th} = 32V.}$

– In order to find $R_{Th}$, we short circuit points a and b, and find $i_{sc}$.



KCL at node 1:

$$\frac{V_1 - 25}{5} + \frac{V_1}{20} - 3 + i_{sc} = 0 \text{ ——— } (1)$$

Also, $V_1$ is the voltage across the 4Ω resistor.

$\Rightarrow V_1 = 4\, i_{sc}$ ——(2) (Ohm's law across the 4Ω resistor)

Thus, (1) and (2) can be solved simultaneously and

$$\underset{(4)}{\frac{V_1 - 25}{5}} + \underset{(1)}{\frac{V_1}{20}} + \underset{(5)}{\frac{V_1}{4}} = \underset{(20)}{\frac{3}{1}}$$

$4V_1 - 100 + V_1 + 5V_1 = 60$

$10V_1 = 160$

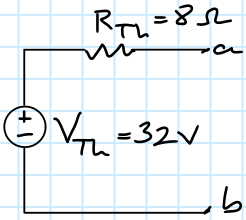
$\Rightarrow V_1 = 16\,V.$

and $i_{sc} = \frac{V_1}{4} = \frac{16}{4} = 4A$ and $\boxed{R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{32}{4} = 8\,\Omega.}$

# P14

Thursday, March 16, 2023 9:52 AM

Thus, the Thévenin equivalent circuit is:



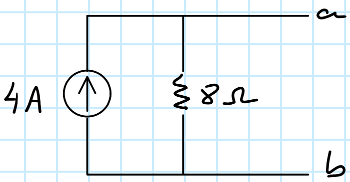
In order to obtain the "Norton Equivalent Circuit", we can make "source transformation", where

$$\boxed{R_{Th}\Big|_{\text{Thevenin}} = R_{th}\Big|_{\text{Norton}}}$$

The current source of Norton circuit is the short circuit current, that is

$$\boxed{i_{sc} = \frac{V_{Th}}{R_{Th}} = \frac{32}{8} = 4A}$$

Then, for this example, the Norton equivalent circuit is



(Norton equivalent).

Alternative technique to find Thévenin equivalent resistance:

If there are only independent sources, to find the Thévenin equivalent resistance:

- Short circuit voltage sources and
- Open circuit current sources.

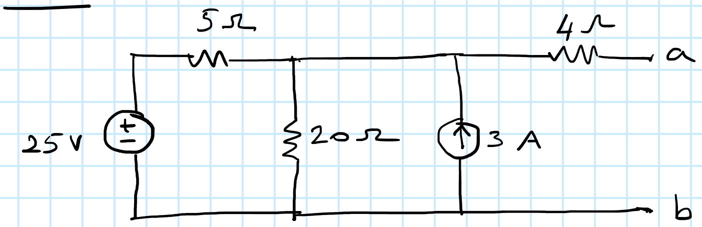
} Deactivating the indep sources

- Then, evaluate the resistance $R_{Th}$ wrt a and b.

# P15

Thursday, March 16, 2023 10:03 AM

**Ex:**



Find $R_{Th} = ?$ w.r.t points a and b.

**Ans:**

$$\Rightarrow R_{Th} = 4 + (5 \parallel 20)$$

$$= 4 + \frac{20 \cdot 5}{20+5} = 4 + \frac{20\cdot\cancel{5}}{\underset{5}{\cancel{25}}}$$

$$= 4 + 4 = 8\,\Omega.$$

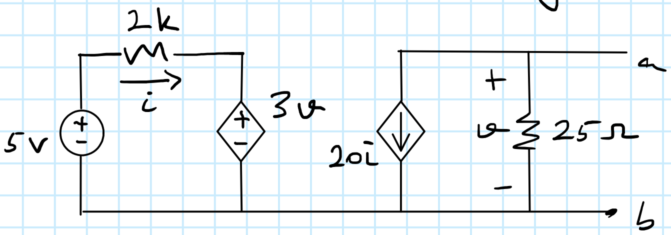
## Another alternative technique to find The Thévenin resistance:

If the circuit contains dependent sources:

- First, deactivate all indep. sources
- Apply a test voltage "$V_T$" btw. the terminals a and b.
- $R_{Th} = \frac{V_T}{i_T}$ , where $i_T$ is the current passing through $V_T$.

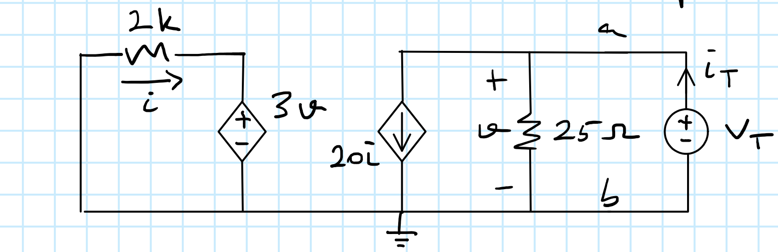
**Ex:**

Find the Thévenin resistance $R_{Th}$ for the circuit below using the method that was just described.



**Ans:**

- First, we de-activate all indep. sources:

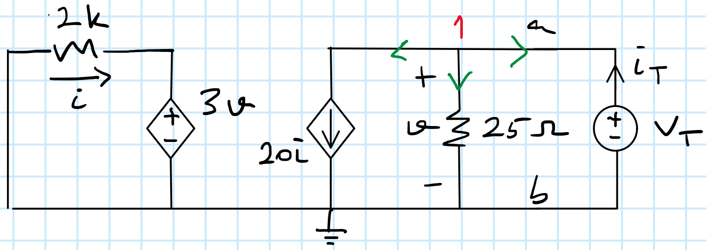


- Apply the test voltage $U_T$

# P16

Thursday, March 16, 2023 10:33 AM

- We need to find $\frac{V_T}{i_T} = R_{Th}$. Let us use the node-voltage method:



KcL at node 1:

$$20i + \frac{v}{25} = i_T \quad —(1)$$

where $i = \frac{-3v}{2k} = \boxed{\frac{-3V_T}{2000}}$ (since $v = V_T$)

Then,

$$20\cdot\left(\frac{-3V_T}{2000}\right) + \frac{V_T}{25} = i_T \quad —(2)$$

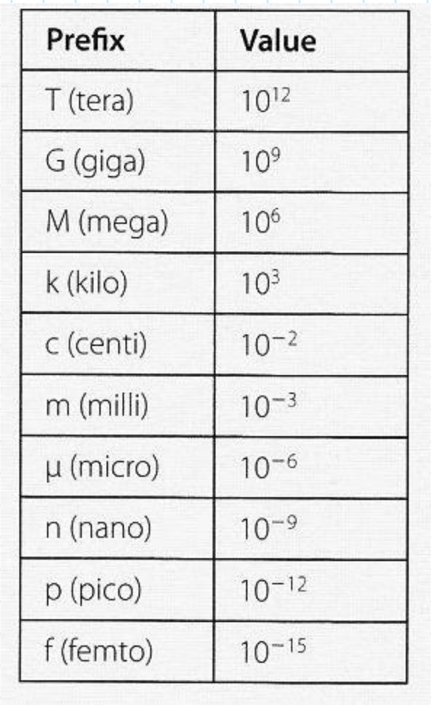
Solve equation (2) for $\frac{V_T}{i_T}$.

$$\underset{(1)}{\frac{-3V_T}{100}} + \underset{(4)}{\frac{V_T}{25}} = i_T$$

$$-3V_T + 4V_T = 100\, i_T$$

$$V_T = 100\, i_T$$

$$\Rightarrow R_{Th} = \frac{V_T}{i_T} = 100\Omega.$$



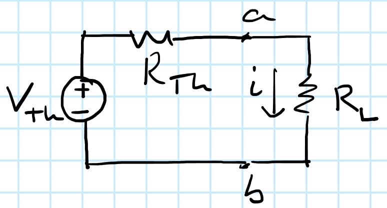
| Prefix | Value |
|---|---|
| T (tera) | $10^{12}$ |
| G (giga) | $10^{9}$ |
| M (mega) | $10^{6}$ |
| k (kilo) | $10^{3}$ |
| c (centi) | $10^{-2}$ |
| m (milli) | $10^{-3}$ |
| µ (micro) | $10^{-6}$ |
| n (nano) | $10^{-9}$ |
| p (pico) | $10^{-12}$ |
| f (femto) | $10^{-15}$ |

# P17

Thursday, March 16, 2023 10:40 AM

## Maximum Power Transfer:

Consider the following circuit:



– The power consumed by the resistor $R_L$ is:

$$P_{R_L} = i^2 . R_L \text{ (w)}$$

– We want to find a value for $R_L$ such that $P_{R_L}$ is maximum. This is called "maximum power transfer".

– The power consumed by $R_L$, $P_{R_L}$, is a function of $R_L$.

$$\Rightarrow P_{R_L}(R_L) = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 . R_L$$

To find the max. $P_{R_L}(R_L)$

$$\frac{d P_{R_L}(R_L)}{d R_L} = 0 .$$

$$2\left(\frac{V_{Th}}{R_{Th}+R_L}\right) \cdot \left[\frac{-V_{Th}}{(R_{Th}+R_L)^2}\right] R_L + \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 = 0$$

Re-arange the equation,

$$2\cancel{\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2} \cdot \frac{1}{(R_{Th}+R_L)} \cdot R_L = \cancel{\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2}$$

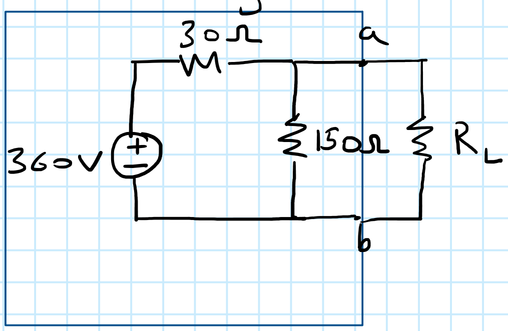
$$2\cancel{R_L} = R_{Th} + \cancel{R_L}$$

$$\Rightarrow \boxed{R_L = R_{Th}}$$ (Condition for max. power transfer).

$$\boxed{P_{R_L} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \frac{V_{Th}^2}{4R_L^{\cancel{2}}}\cdot \cancel{R_L} = \frac{V_{Th}^2}{4R_L}}$$ (W) (Power consumed by $R_L$ when $R_L = R_{Th}$.)

Ex:

For the given circuit below:

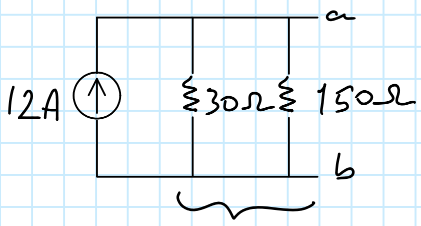


i) Find $R_L$ for max. power transfer

ii) Find $P_{R_L}\Big|_{max}$.

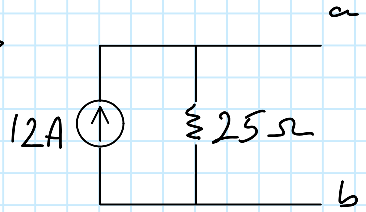
– First, find the Thévenin equivalent circuit for the circuit inside the blue box.

Using the source transformation

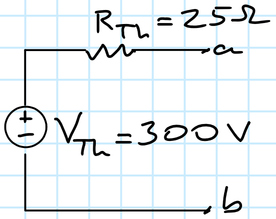


$$(30 \| 150) = \frac{150\times30}{150+30} = \frac{150\times3\cancel{0}}{{}_6\,\cancel{18}\cancel{0}} = 25\,\Omega$$

⇒



$\xrightarrow{\text{Source Transfr.}}$



Thévenin equivalent Circuit

Then, for max. power transfer $R_L = R_{Th} = 25\,\Omega$.

and $$P_{R_L} = \frac{V_{Th}^2}{4R_L} = \frac{300^2}{4\times25} = \frac{3\cancel{00}\cdot300}{1\cancel{00}} = 900\text{ W}.$$

↓HW #4↓

## Complex Numbers

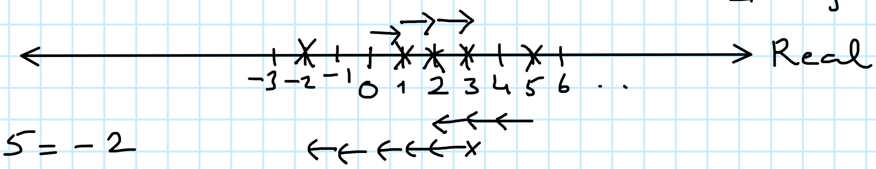
Let us first consider the real numbers

$\mathbb{R} = \{\dots, -1, 0, 1, \dots\}$

### Addition & Subtraction:

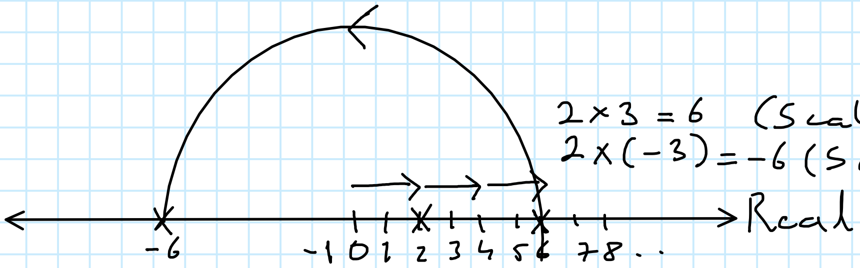
$1+2=?\ 3.$

$5-3=2$ } Translation (motion)



Consider $3-5=-2$

### Multiplication & Division



$2\times3=6$ (Scaling)

$2\times(-3)=-6$ (Scaling + Rotate)

⇒ Multiplication is scaling + rotating (if − numbers)

Also, division is similar It is a (Down scaling + rotation)

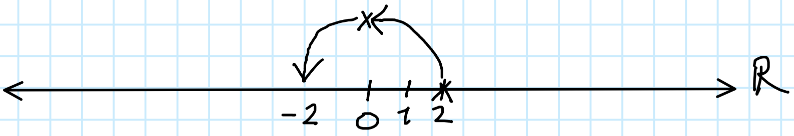
* * *

Can we rotate a quarter of a turn (90°) ?

$2\times z\times z=-2$

$z^2=-1$

$z=\sqrt{-1}$



- Therefore, multiplying any real number by $z=\sqrt{-1}$ rotates the number by a $\frac{1}{4}$ of a turn.

We call this number "Complex number" and use the following notation

$$\boxed{z=i=\sqrt{-1}}$$

or

$$\boxed{z=j=\sqrt{-1}}$$

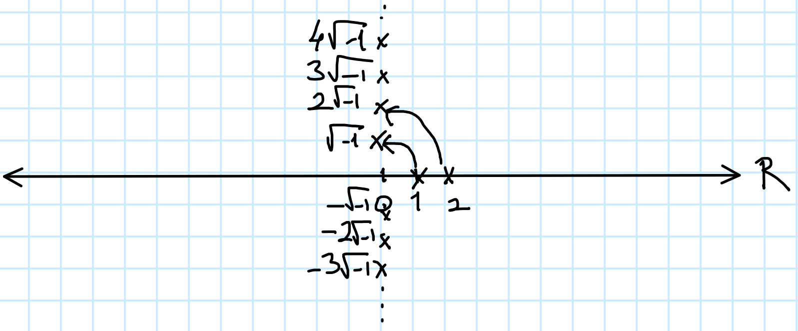
# P20

Friday, March 24, 2023 10:31 AM

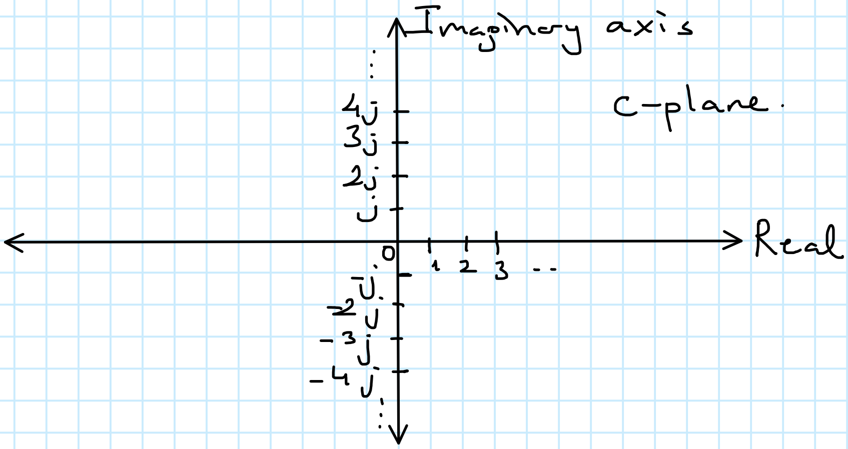
⇒ $z = j = \sqrt{-1}$ , where is this number on the number line

$1\times\sqrt{-1} = \sqrt{-1} = j$

$2\times\sqrt{-1} = 2\sqrt{-1} = 2j$



⇒ We can draw a vertical line.



- This plane is called "complex plane" (c-plane) and any number on this plane is called a "complex number"

Ex

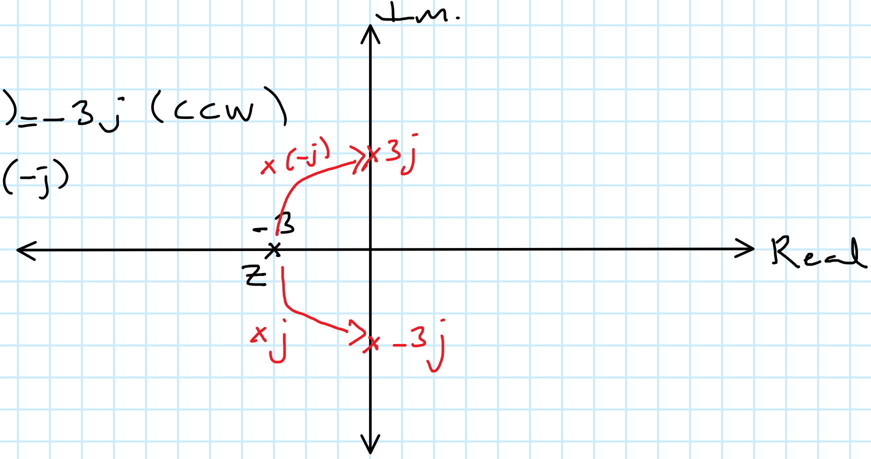
Suppose $z = -3$, what should be done to rotate this number a $\frac{1}{4}$ of a turn cw and ccw?

Ans

$z \times (j) = (-3)\times(j) = -3j$ (ccw)

$z \times (-j) = (-3)\times(-j)$

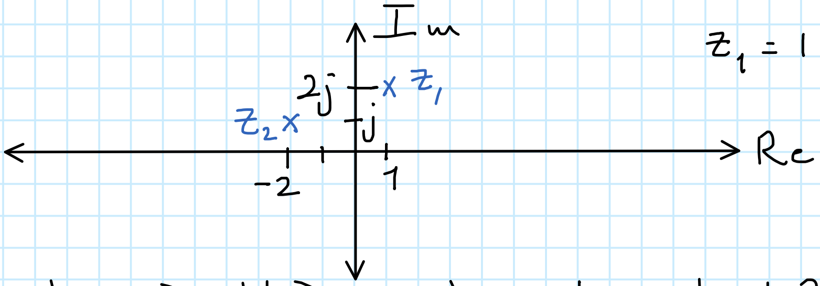
$= 3j$ (cw)



# P21

Friday, March 24, 2023 10:43 AM

Writing a Complex Number in terms of its Real and imaginary parts (components):



$z_1 = 1 + 2j$

$z_2 = z_1 \times j = ?$ where is this number located?

$j = \sqrt{-1}$

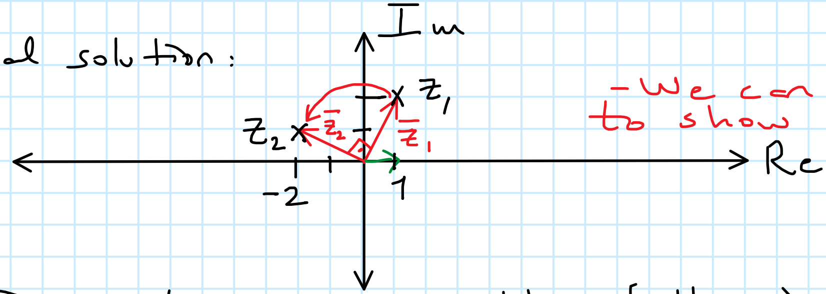
$j^2 = -1$

Ans

Algebraic solution: $z_1 \times j = (1 + 2j) \times (j) = j + 2j^2$

$= -2 + j$

Graphical solution:



-We can use a vector to show complex numbers

⇒ Up to this point, we have the following notations:

$z = \underbrace{a}_{\text{Real}} + \underbrace{jb}_{\text{Imaginary.}}$ (component notation)

$\bar{z} = \bar{a} + j\bar{b}$ (vector notation, used in geometric analysis)

—o—

What if we multiply $z_1$ by $2j$.

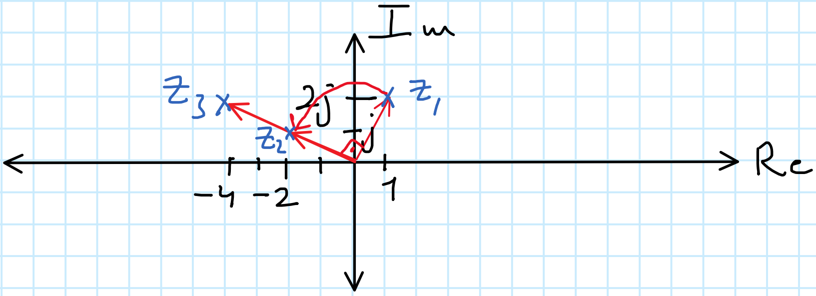
$z_3 = z_1 \times (2j) = ?$

Algebraic Sol.:

$(1 + 2j) \cdot (2j)$

$= 2j + 4j^2$

$z_3 = -4 + 2j$



⇒ $z_1 \times (2j)$ = (rotation + scaling by 2.)

—o—

# P22

Friday, March 24, 2023 10:59 AM

## Power of a Complex Number:

$j^n = ?$

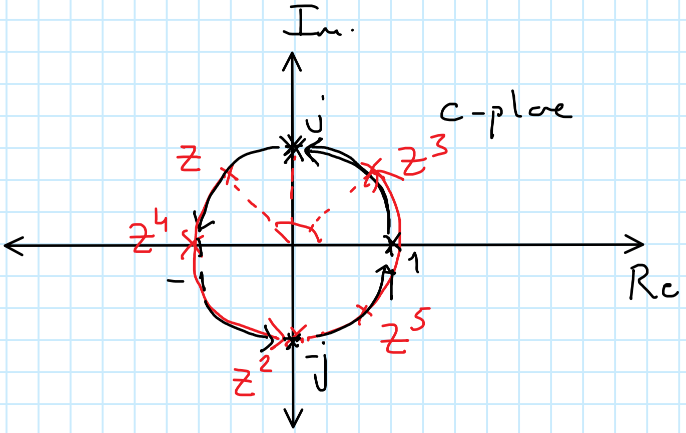
$1 \times j = j$ $(j^1)$

$j \times j = -1$ $(j^2)$

$-1 \times j = -j$ $(j^3)$

$-j \times j = 1$ $(j^4)$

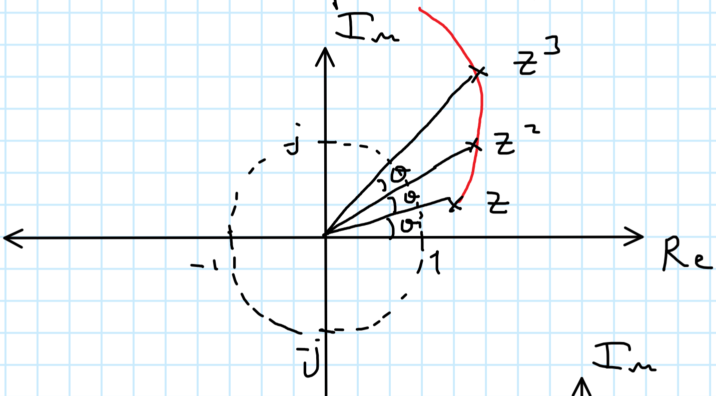
$1 \times j = j$ $(j^5)$



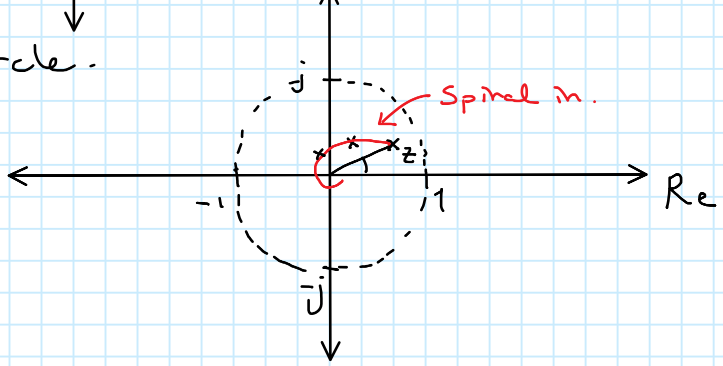
=> Taking the power of (j) rotates ccw on the unit circle

- Taking the power of any complex number z which is on the unit circle rotates the number ccw by the same angle that z makes with the real axis

- What if z is outside the unit circle? Then, what happens when we take the power of z



- If z is inside the unit circle.

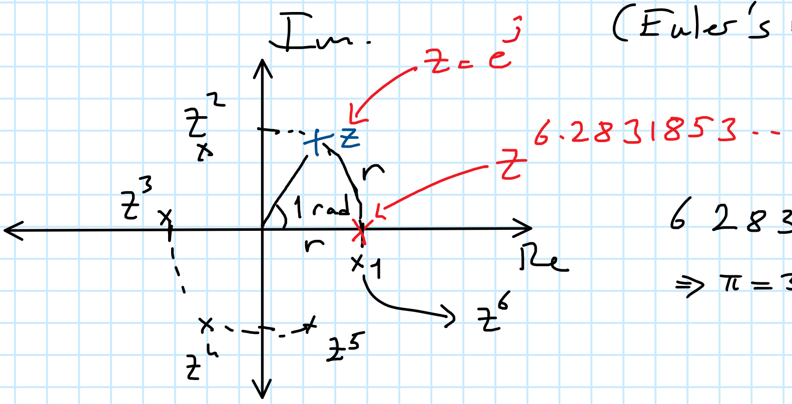


# P23

Friday, March 24, 2023 11:03 AM

## Complex Exponential:

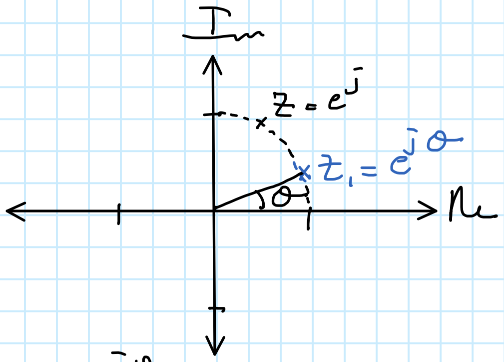
$z = e^{j\theta}$ (complex exponential), $e = 2.71828\ldots$ (Euler's number)



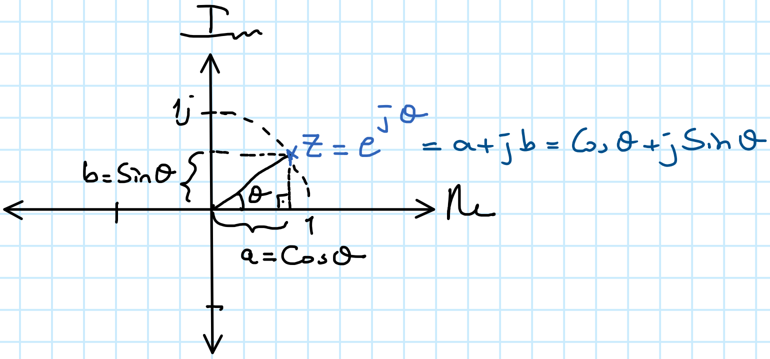
$6.2831 = 2\pi$

$\Rightarrow \pi = 3.1416\ldots$

– Also, $z_1 = e^{j\theta}$ is on the unit circle with angle $\theta$ in radians

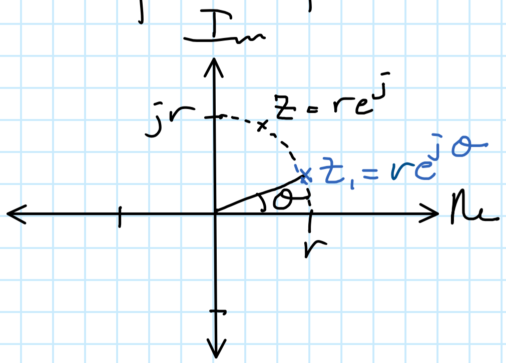


Let us observe $z = e^{j\theta}$ closely:

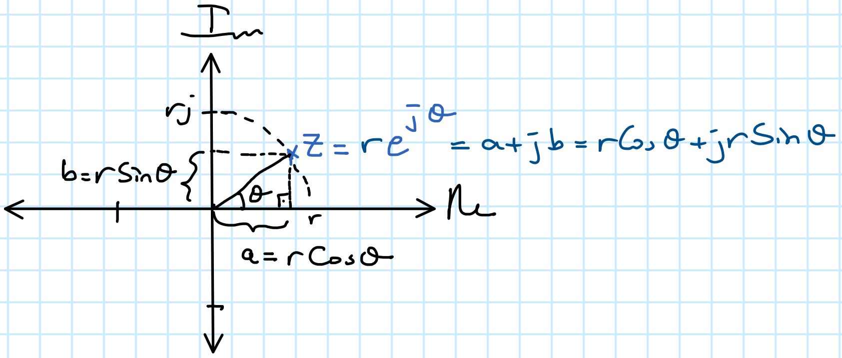


$$\Rightarrow \underbrace{z = e^{j\theta}}_{\text{Exponential notation}} = \underbrace{a + jb = \cos\theta + j\sin\theta}_{\text{Rectangular notation}}$$

Let us analyze the complex exponential $z = re^{j\theta}$ where is a constant.



- $z_1 = re^{j\theta}$ is on the circle whose radius is A.
- Also, let us observe $z = re^{j\theta}$ closely:



$$\Rightarrow \boxed{\underbrace{z = re^{j\theta}}_{\text{Exponential notation}} = \underbrace{a+jb}_{\text{Rectangular notation}} = r\cos\theta + jr\sin\theta \text{ (Euler's formula)}}$$

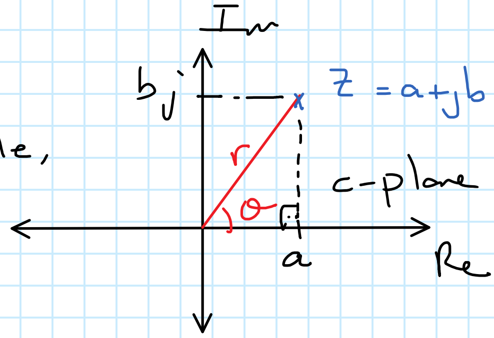
# P25

Friday, March 24, 2023 11:40 AM

Until now, The formats for complex numbers.

$z = a+jb$ = Rectangular , $\bar{z}$ = vectorial,

3rd format. exponential form (Polar form)



$z = a+jb = re^{j\theta} = r\angle\theta$

where

r = Magnitude, amplitude, radius, modulus.

θ = Argument, Angle, Phase.

Rectangul to Polar Form Transformation:

$\boxed{r^2 = a^2+b^2}$ , $\boxed{\theta = \tan^{-1}\frac{b}{a}}$

Polar to Rectangular Transformation:

$$\boxed{\begin{array}{l} a = r\cos\theta \\ b = r\sin\theta \end{array}}$$

Ex:

Given $z = 3+j4$, find its polar representation.

Ans:

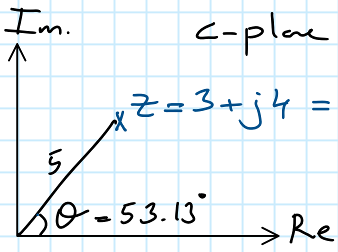
$a = 3$, $b = 4$

$\Rightarrow r = \sqrt{a^2+b^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5.$

and $\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{4}{3} = 0.9273$ rad. $= 53.13°$.

$\Rightarrow z = 5e^{j0.9273}$

$= 5e^{j53°} = 5\angle 53°$



$z = 3+j4 = 5e^{j0.9273}$

# P26

Friday, March 24, 2023 11:44 AM

## Complex Algebra:

### Addition & Subtraction of Complex Numbers:

- Using rectangular notation is easier
- Add or subtract the Real and imaginary parts among themselves.

**Ex:**

$z_1 = 3+4j$

$z_2 = 2-2j$

a.) $z_1 + z_2 = ?$

b.) $z_1 - z_2 = ?$

**Ans**

$z_1 + z_2 = (3+4j) + (2-2j)$

$= (3+2) + (4j - 2j)$

$= 5 + 2j$

$z_1 - z_2 = (3+4j) - (2-2j)$

$= (3+4j) - 2 + 2j$

$= (3-2) + (4j + 2j)$

$= 1 + 6j$

### Multiplication & Division

- Polar form is easier.
- Multiply the magnitudes and sum the angles.

**Ex**

$z_1 = 3+4j$, $z_2 = 8-6j$

a.) $z_1 \cdot z_2 = ?$

b.) $\frac{z_2}{z_1} = ?$

**Ans**

$z_1 \cdot z_2 = (3+4j) \cdot (8-6j)$

$= 24 - 18j + 32j - 24j^2$

$= 24 - 18j + 32j + 24$

$= 48 + 14j$

# P27

Friday, March 24, 2023 10:51 AM

Alternatively, converting these complex numbers into polar form:

$z_1 = 3+4j = 5e^{j0.93}$,

$z_2 = 8-6j = \sqrt{8^2+(-6)^2}\, e^{j\tan^{-1}\left(\frac{-6}{8}\right)} = 10e^{-j0.64}$

$\Rightarrow z_1 . z_2 = 5e^{j0.93} . 10e^{-j0.64} = 50e^{j(0.93-0.64)} = 50e^{j0.29}$

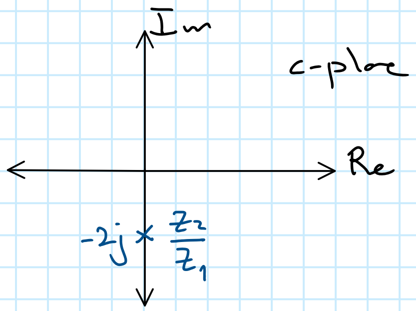
Using the Euler's theorem: $z_1 . z_2 = 50\cos 0.29 + j50\sin 0.29$

$= 48 + j14$. which is the same as before.

$$\frac{z_2}{z_1} = \frac{10e^{-j0.64}}{5e^{j0.93}} = 2 . e^{-j0.64-j0.93} = 2e^{-j\underbrace{1.57}_{\frac{\pi}{2}}} = 2e^{-j\frac{\pi}{2}} = -2j$$

$\Rightarrow r = 2$

$\theta = -\frac{\pi}{2}$



↓HW#5↓

- Why do we need complex numbers?

Ans: Mathematical Computations with sin or cos functions are difficult. To make them easier, we use complex numbers.

Ex:

Given $f_1(t) = \sin(4\pi t + \frac{\pi}{2})$, $f_2(t) = \sin(4\pi t + \frac{3\pi}{2})$

$f_1(t) \times f_2(t) = ?$

Ans:

This computation is not very easy.

First, we need the trigonometric Identity

$$\sin a \times \sin b = -\frac{1}{2}\left[\cos(a+b) - \cos(a-b)\right]$$

# P28

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Using this identity, we get

$$\sin(4\pi t+\frac{\pi}{2})\times\sin(4\pi t+\frac{3\pi}{2})=\frac{-1}{2}\left[\cos(8\pi t+2\pi)-\cos(-\pi)\right]$$

$$=\frac{-1}{2}\left[\cos(8\pi t)-(-1)\right]$$

$$=-0.5\cos(8\pi t)-0.5$$

Similarly, if we wanted to find $\frac{f_1(t)}{f_2(t)}$, or any other algebraic expression, it is not so easy to find the solution.

## The sinusoidal Function:

$$V(t)=V_m\cos(\underbrace{\omega t+\phi}_{\theta\,=\,\text{Angle (rad)}})$$

$t$ = time,

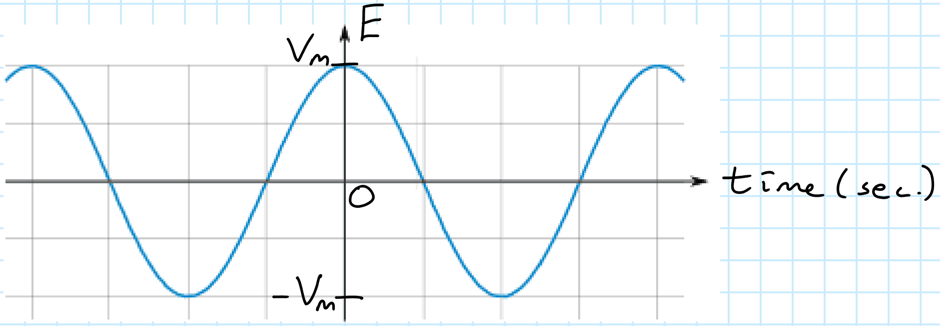
$V_m$ = Amplitude,

$\omega = 2\pi f$ = Radian freq.

$\phi$ = Phase (rad.),

## Amplitude:

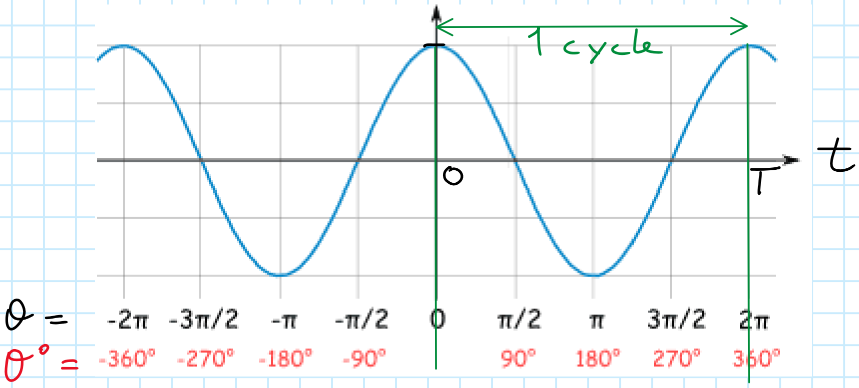
$$V(t)=V_m\cos(\omega t+\phi)$$



## Radian Frequency ($\omega$):

Let us consider $\phi = 0$ for simplicity, then we have

$$V = V_m \cos(\omega t)$$

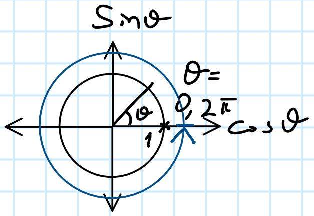


When $t = 0$, $V = V_m$, $\theta = 0$

When $t = T$, $V = V_m$, $\theta = 2\pi$

$\Rightarrow \omega T = 2\pi$

$\boxed{\omega = \frac{2\pi}{T}}$ $\left(\frac{rad}{sec}\right)$



where

$T$ = Time for 1 turn of $\theta$ or time for 1 cycle

Then, $\omega$ = number of $2\pi$ radians in 1 sec.

$\omega = 2\pi$ means there is $2\pi$ rad in 1 sec.

$\omega = 4\pi$ means there are $2 \times 2\pi$ rad. (2 turns around the unit circle)

$\omega = 6\pi$ means there are $\underbrace{3}_{f} \times 2\pi$ rad (3 turns " " " " )

⋮

$\Rightarrow$ $\boxed{\omega = 2\pi f}$ where $f$ = number of turns in 1 sec.

and $\boxed{T = \frac{1}{f}}$.

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Ex:

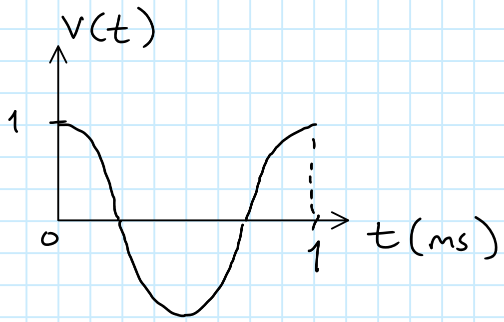
$V(t) = \cos(2000\pi t)$ is given. Find the Hertz frequency f? Also find T, and draw v(t).

Ans:

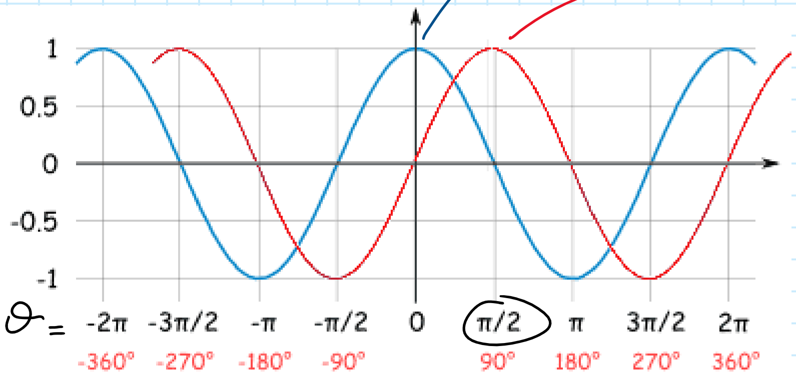
$\omega = 2000\pi \ \frac{rad}{sec}$ , $T = \frac{1}{f} = \frac{1}{1000} = 1 \times 10^{-3} = 1 ms.$

$2000\pi = 2\pi f$

$\Rightarrow f = 1000 \ Hz = 1 \ kHz.$



Phase ( $\phi$ ) :



$E = \cos(\omega t)$

$E = \cos(\omega t - \frac{\pi}{2})$

$= \sin(\omega t)$

Thus, the phase $\phi$ makes the cos function shift right or left depending on its sign. "+" shifts left and "−" shifts right.

# P31

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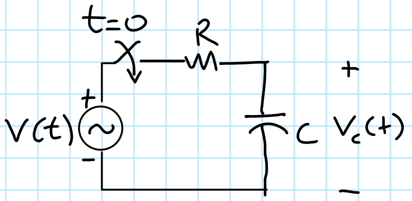
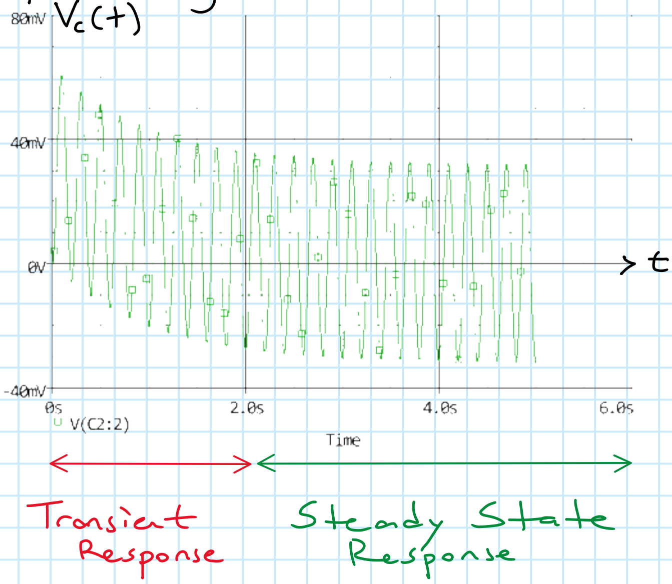
## – Sinusoidal State State Response –

Sinusoidal means sine or cosine functions.

Steady state means that the circuit's initial response has finished, and everything has been settled.

For example, consider the following circuit





Transient Response: Time interval for all capacitor and/or inductors to charge or discharge.

Steady State: The time after everything is settled down.

To find the steady state response of a given circuit with sinusoidal sources, we use "Phasors" as follows.

$$v(t) = V_m \cos(\omega t + \phi) \xrightarrow{\text{Phasor Transform}} \bar{V} = V_m e^{j\phi} \text{ (Phasor)}$$

(Time expression)

– The major difference in phasor expression is that we exclude the "$\omega t$" term. Thus, the circuit must have the same frequency. This implies linearity.

$$\bar{V} = V_m e^{j\phi} \xrightarrow{\text{Inverse Phasor Transform}} v(t) = \mathrm{Re}[\bar{V} \times e^{j\omega t}]$$

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If we use the "sine" function as the reference, then

$$V(t) = V_m \text{Sin}(\omega t + \phi) \xrightarrow{\text{Phasor Transform}} \bar{V} = V_m e^{j\phi} \text{ (Phasor)}$$

(Time expression)

$$\bar{V} = V_m e^{j\phi} \xrightarrow[\text{Phasor Transform}]{\text{Inverse}} V(t) = \text{Im}\left[\bar{V} \times e^{j\omega t}\right]$$

Ex:

Given $f_1(t) = \text{Sin}(4\pi t + \frac{\pi}{2})$, $f_2(t) = \text{Sin}(4\pi t + \frac{3\pi}{2})$

$f_1(t) \times f_2(t) = ?$ using phasors.

Ans:

When we multiply $f_1(t)$ by $f_2(t)$, the frequency is doubled. In this case $\omega t = 4\pi$, however $\omega t = 8\pi$ for $f_1(t) \cdot f_2(t)$. Thus, this is a non-linear system and can not be solved by phasors.

Ex:

Given $f_1(t) = \text{Sin}(4\pi t + \frac{\pi}{2})$, $f_2(t) = \text{Sin}(4\pi t)$

$y_1(t) = f_1(t) + f_2(t) = ?$ using phasors.

Addition of sinusoidals is a linear operation, thus we can use phasors.

Ans:

$$\bar{f}_1 = 1 \cdot e^{j\frac{\pi}{2}} = e^{j\frac{\pi}{2}} = \cancelto{0}{\cos\frac{\pi}{2}} + j\sin\frac{\pi}{2} = j$$

$$\bar{f}_2 = 1 \cdot e^{j0} = 1$$

$$\Rightarrow \bar{y}_1 = \bar{f}_1 + \bar{f}_2 = 1 + j = \sqrt{2}\, e^{j\tan^{-1} 1} = \sqrt{2}\, e^{j0.78} = \sqrt{2}\angle 0.78 = \sqrt{2}\angle 45^\circ$$

$$y_1(t) = \text{Im}\left[\bar{y} \cdot e^{j4\pi t}\right] = \text{Im}\left[\sqrt{2}\, e^{j0.78} \cdot e^{j4\pi t}\right]$$

$$= \sqrt{2}\, \text{Sin}(4\pi t + 0.78)$$

# P33

Monday, March 13, 2023 11:58 AM

Ex:

$$\sin(2000\pi t) + \cos(2000\pi t) = ?$$

Ans:

$$y(t) = \cos\left(2000\pi t - \frac{\pi}{2}\right) + \cos(2000\pi t)$$

In phasor domain,

$$\bar{y} = \underbrace{e^{-j\frac{\pi}{2}}}_{-j} + \underbrace{e^{j0}}_{1} = 1 - j$$

Converting to a polar form

$$\bar{y} = \sqrt{2}\, e^{j\tan^{-1} -1} = \sqrt{2}\, e^{-j0.78}$$

Then

$$y(t) = \mathrm{Re}\left[\bar{y} \cdot e^{j\omega t}\right] = \mathrm{Re}\left[\sqrt{2}\, e^{-j0.78} \cdot e^{j2000\pi t}\right]$$

$$= \sqrt{2}\cos(2000\pi t - 0.78)$$

* * *

- In circuit theory, all equations (Ohm's law, KVL, KCL) of circuits having sinusoidal sources are linear and can be solved by phasors.

# P34

Thursday, April 6, 2023 9:43 AM

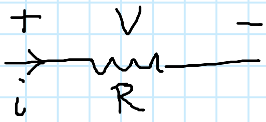
↓HW#6↓

## - Sinusoidal Steady State Response -

## - Passive Circuit Elements (R, L, C) in Frequency Domain -

$\underbrace{\text{Frequency Domain}}_{\text{Phasor domain}}$

### 1-) V-I Relation for a Resistor:



$$v = R[I_m \cos(\omega t + \phi)]$$
$$= RI_m \cos(\omega t + \phi) \ (V)$$

$$\Rightarrow \bar{V} = RI_m \cdot e^{j\phi} = \underbrace{RI_m\angle\phi}_{\bar{I}}$$

or $\boxed{\bar{V} = R\bar{I}}$

### 2-) V-I Relation for an Inductor.

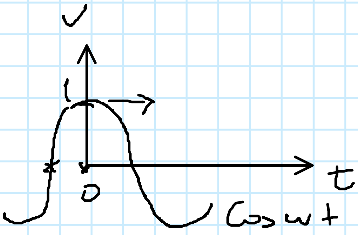
$v = L\dfrac{di}{dt}$ assuming that $i = i(t) = I_m \cos(\omega t + \phi)$

or

$$V = L(-\omega) I_m \sin(\omega t + \phi)$$
$$= -\omega L I_m \sin(\omega t + \phi)$$

or

$$V = -\omega L I_m \cos(\omega t + \underbrace{\phi - \tfrac{\pi}{2}}_{\text{Phase}})$$



Thus,

$$\bar{V} = -\omega L I_m \cdot e^{j(\phi - \frac{\pi}{2})}$$

and $\bar{V} = -\omega L I_m e^{j\phi} \underbrace{e^{-j\frac{\pi}{2}}}_{-j} = j\omega L \underbrace{I_m e^{j\phi}}_{=\bar{I}}$

Thus, $\boxed{\bar{V} = j\omega L\bar{I}}$ $\Rightarrow v(t) = L\dfrac{di(t)}{dt} \xrightarrow{P} \bar{V} = j\omega L\bar{I}$

$$\Rightarrow \frac{d}{dt} \xrightarrow{P} j\omega$$

# P35

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## 3-) V–I Relation for a Capacitor:

For a capacitor,

$$i = C \cdot \frac{dv}{dt} \quad , \text{ where } \quad v = V_m \cos(\omega t + \phi)$$

Then,

$$i = -C\omega V_m \sin(\omega t + \phi)$$

or $$i = -C\omega V_m \cos(\omega t + \phi - \frac{\pi}{2})$$

$$\vdots$$

$$\Rightarrow \boxed{\bar{I} = j\omega C \bar{V}} \quad \text{or} \quad \boxed{\bar{V} = \frac{\bar{I}}{j\omega C} = -j\frac{\bar{I}}{\omega C}}$$

## Impedance & Reactance:

Define $\boxed{\bar{V} = Z\bar{I}}$ where $Z$ is called "impedence"

$\boxed{Z = R + jX}$ is a complex number.

where $R$ = Resistance (real part of the impedance)

$X$ = Reactance (imag. " " " " ).

Thus, for passive elements (R, L, C):

| | $Z(\Omega)$ | $R$ | $X$ |
|---|---|---|---|
| Resistor: | $R$ | $R$ | $0$ |
| Inductor: | $j\omega L$ | $0$ | $\omega L$ |
| Capacitor: | $\frac{-j}{\omega C}$ | $0$ | $\frac{-1}{\omega C}$ |

Thus, $Z$ : impedance is the resistance for AC circuits.

Also, define

$$\boxed{Y = \text{Admittance} = \frac{1}{Z}} \quad (\mho = \text{mho's or } S\text{: Siemens}).$$

In addition, $Y = G + jB$

where $G$ = Conductance

$B$ = Susceptance.

# P36

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## - Kirchoff's Laws in Frequency Domain -

1-) KVL in Phasor domain:

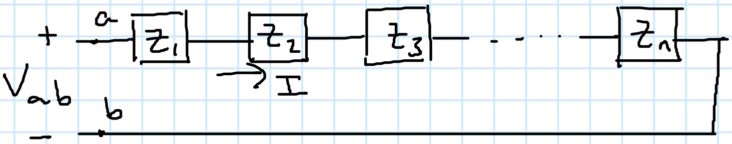
$$\bar{V}_1 + \bar{V}_2 + \cdots + \bar{V}_n = 0$$

2-) KCL in Phasor domain:

$$\bar{I}_1 + \bar{I}_2 + \cdots + \bar{I}_n = 0$$

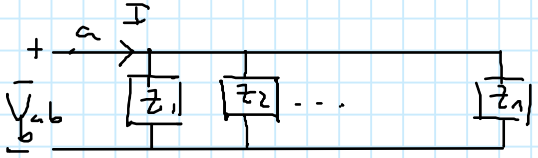
3-) Series and Parallel Connections:

a-) Series Connection



$$Z_{ab} = \frac{\bar{V}_{ab}}{\bar{I}} = Z_1 + Z_2 + \cdots + Z_n$$

b-) Parallel Connection:



$$Z_{ab} = \frac{\bar{V}_{ab}}{\bar{I}} \Rightarrow$$

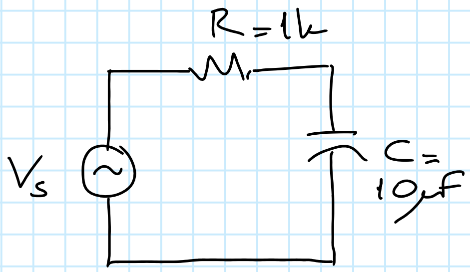
$$\frac{1}{Z_{ab}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

However,

$Y_{ab}$ = Admittance = $G + jB$

$Y_{ab} = Y_1 + Y_2 + \cdots + Y_n$

Ex:



+

$V_c(t) = ?$ (Steady State)

−

Given $V_s = 10\cos(2000\pi t)$ V.

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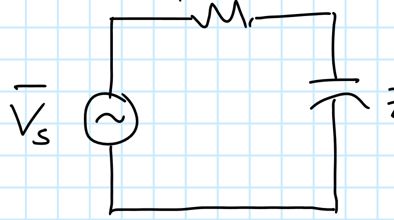
Ans:

Step 1: Transform the circuit into phasor domain

$Z_R = R = 1000\Omega$



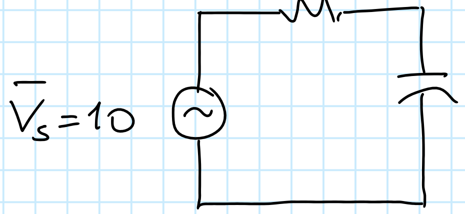
$\bar{V}_s = 10\cancel{e^{j0}} \xrightarrow{} 1 = 10$

$Z_c = \frac{1}{j\omega C}$

$= \frac{-j}{\omega C}$

Step 2: Use the circuit analysis techniques to solve the unknown voltage or current.

$Z_R = R = 1000\Omega$



$Z_c = \frac{1}{j\omega C}$

$= \frac{-j}{\omega C} = \frac{-j}{(2\cancel{000}\pi)(1\cancel{0} \times 10^{\cancel{-6}\,\cancel{-5}\,-2})} = \frac{-j}{2\pi \times 10^{-2}}$

$= \frac{-50j}{\pi}$

From the voltage division:

$\bar{V}_c = \bar{V}_s \cdot \frac{Z_c}{Z_c + Z_R} = 10 \cdot \frac{-50j/\pi}{\frac{-50j}{\pi} + 1000} = 10 \cdot \frac{\frac{-50j}{\cancel{\pi}}}{\frac{1000\pi - 50j}{\cancel{\pi}}} =$

$\bar{V}_c = 10 \cdot \frac{\overset{-j}{-5\cancel{0}j}}{\underset{20\pi}{10\cancel{00}\pi} - \underset{-j}{5\cancel{0}j}} = 10 \cdot \frac{-j}{\boxed{20\pi - j}} =$

$\hookrightarrow \approx 60 - j = \sqrt{60^2+1}\, e^{j \cancel{\tan^{-1}} \frac{-1}{60}} \xrightarrow{} 0$

$\approx 60 e^{j0} = 60.$

$= \cancel{1}\cancel{0} \cdot \frac{-j}{\underset{6}{6\cancel{0}}} = \frac{1}{6}(-j) \approx 0.17 \underbrace{(-j)}_{e^{-j\frac{\pi}{2}}}$

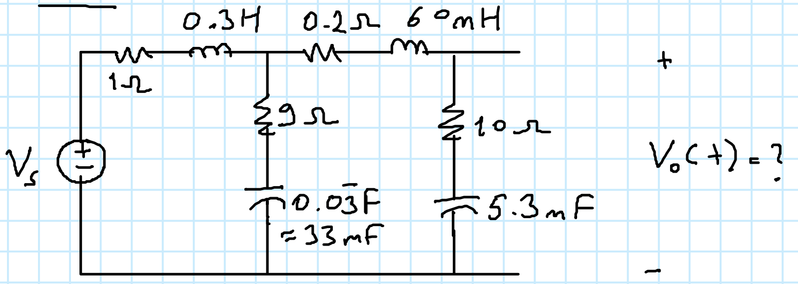
$\Rightarrow \bar{V}_c = 0.17 e^{-j\frac{\pi}{2}}$

$v_c(t) = \mathrm{Re}[\bar{V}_c \cdot e^{j\omega t}] = 0.17\cos(2000\pi t - \frac{\pi}{2})$ V.

# P38

Thursday, April 13, 2023 9:31 AM

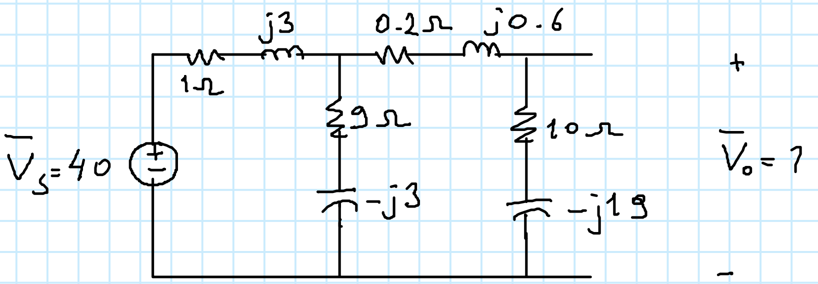
Ex:



$V_s = 40\cos(10t)\ (V)$

Ans:

Phasor domain equivalent circuit is:



$Z_{0.3H} = j\omega L = j(10)(0.3) = j3(\Omega)$

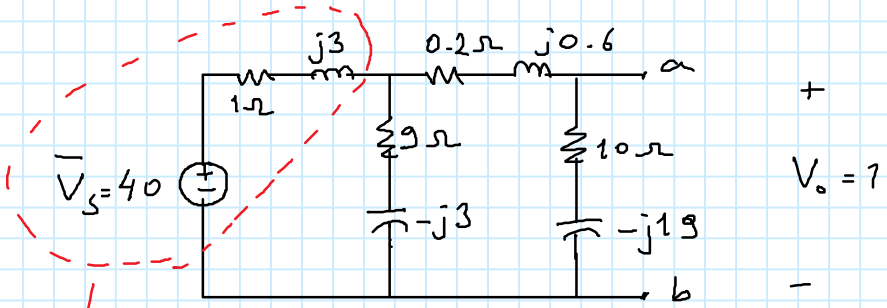
$Z_{33mF} = \frac{-j}{\omega C} = \frac{-j}{10.(0.0\bar{3})} = -j3(\Omega)$

$Z_{60mH} = j\omega L = j(10)(60\times10^{-3})$

$= j0.6\ (\Omega).$

$Z_{5.3mF} = \frac{-j}{\omega C} = \frac{-j}{(10)(5.3\times10^{-3})}$

$= -j19\ (\Omega).$

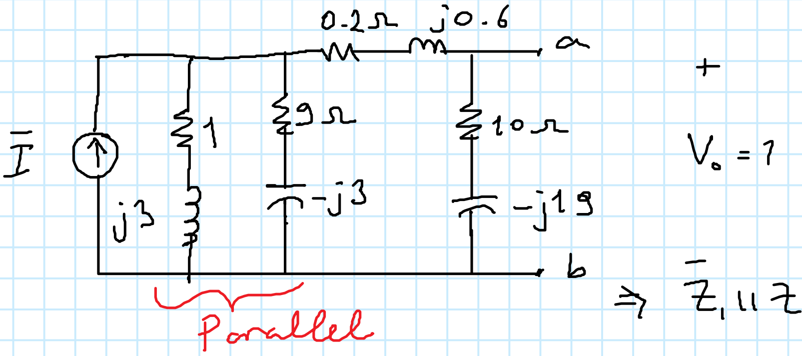


Replace by its Norton equivalent $\Rightarrow \bar{I} = \frac{40}{1+j3} = \frac{40(1-j3)}{10}$

$(1-j3)$

$\bar{I} = 4 - j12\ (A)$

Then, we have



$z_1 = 1+j3\ \Omega$ $z_2 = 9-j3\ \Omega$

$\Rightarrow \overline{Z_1 \parallel Z_2} = Z_t$

$\Rightarrow Z_T = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{(1+j3)(9-j3)}{10}$

$= 1.8+j2.4\ \Omega$

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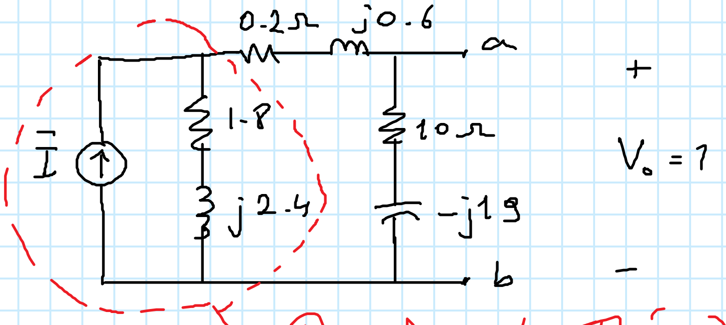
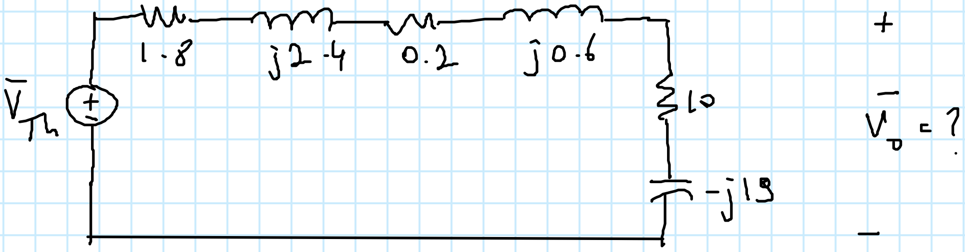
Now, we have:



Transform to Thevenin equivalent

$$\bar{V}_{Th} = (4-j12)(1.8+j2.4) = 36-j12 \ (V)$$

Then,



Using the voltage divider:

$$\bar{V}_o = \bar{V}_{Th} \cdot \frac{(10-j13)}{12-j16} = 36.12 - j18.84 \text{ V}.$$

$$= 40.74 e^{-j0.48} \ (V).$$

Time expression of $\bar{V}_o$:

$$v_o(t) = Re\left[\bar{V}_o . e^{j\omega t}\right]$$

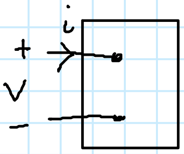
$$= 40.74 \cos(10t - 0.48) \ (V)$$

↓ HW 7 ↓

## - Sinusoidal Steady State Power Calculations - (Chp. 10)

Instantenous power = The power at any instant of time.

$\boxed{p = v \cdot i}$ for the circuit



where $\boxed{v = V_m \cos(\omega t + \theta_v)}$ and $\boxed{i = I_m \cos(\omega t + \theta_i)}$

-If we set our start time such that the system requires shift of both the voltage and current by "$\theta_i$", then:

$$V = V_m \cos(\omega t + \theta_v - \theta_i) \text{ and } i = I_m \cos(\omega t + \cancel{\theta_i} - \cancel{\theta_i})$$

$$\text{or } i = I_m \cos(\omega t)$$

Then, instantenous power becomes.

$$p = v \cdot i = V_m \cos(\omega t + \theta_v - \theta_i) \cdot I_m \cos(\omega t) \quad (W)$$

Then, using the Identity,

$$\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\Rightarrow \quad p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v - \theta_i)$$

Now, we use the Identity: $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\Rightarrow$

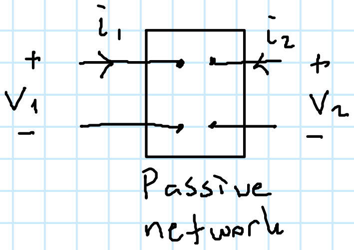
$$\boxed{p = \underbrace{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)}_{P_{avg}} + \underbrace{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos(2\omega t)}_{P_{avg} \cos(2\omega t)} - \underbrace{\frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \cdot \sin(2\omega t)}_{Q \sin(2\omega t)}}$$

—o—

# P41

Thursday, April 13, 2023 10:22 AM

**Passive Network:** one that contains resistors, capacitors, and/or inductors. Passive networks have a power gain less than 1.



$\frac{V_2}{V_1}$ = Voltage gain = $G_v$

$\frac{i_2}{i_1}$ = Current gain = $G_i$

⇒ $P_1 = V_1 \cdot i_1$ = input power

and $P_2 = V_2 \cdot i_2$ = output power

Then, $\frac{P_2}{P_1}$ = Power gain = $G$ , $0 < G < 1$

**Active network:** is the one that $G > 1$. Thus, it contains amplifiers, or sources

For passive circuits (networks).

- When $p > 0$ ⇒ this means the circuit is consuming power
- When $p < 0$ ⇒ this means that the capacitors and/or the inductors give their stored energy into the circuit.

P42

Thursday, April 13, 2023 10:29 AM

## Real (average) and Reactive Power

Real power or average power is given as:

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)}$$ ($1^{st}$ term of p)

and

$$\boxed{Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)}$$ is defined as the "reactive power"

Thus, the instentenous power can be re-written as:

$$\boxed{p = P_{avg} + P_{avg} \cos(2\omega t) - Q \cdot \sin(2\omega t)}$$

## Power for Purely Resistive Circuits:

In these circuits, we have only resistors. Therefore, $\boxed{\theta_v = \theta_i}$, and

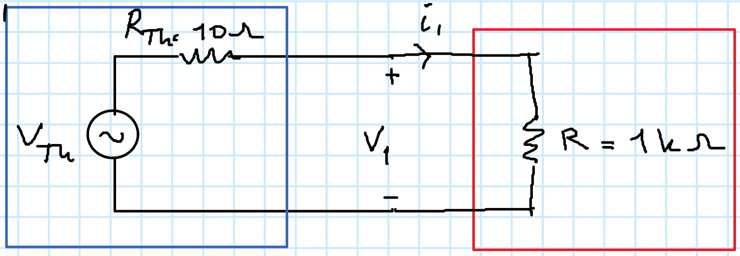
$$p = P_{avg} + P_{avg} \cos(2\omega t)$$ (instentenous power for purely resistive circuits.)

Ex:

$V_{Th} = 10 \cos(\omega t)$

$\omega = 2000\pi \left(\frac{rad}{sec}\right)$.

Thevenin source



Purely resistive circuit (network)

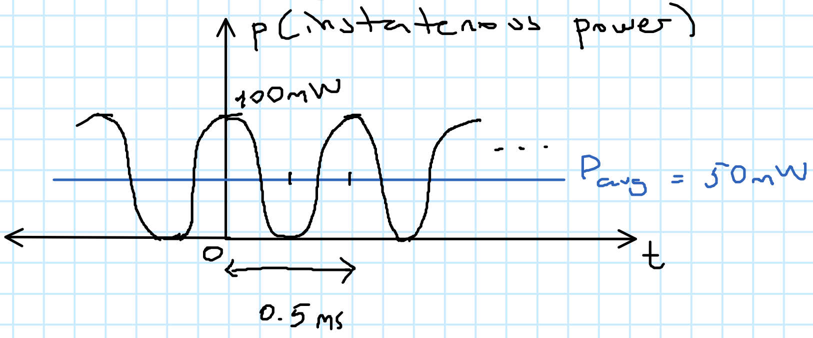
Ans: For this circuit, find $P_{av}$ and p (instentenous power)

$\boxed{V_{Th} = \underset{\sim}{10 \cos(\omega t)}}$, $\Rightarrow$ $\boxed{V_1 \approx V_{Th}}$; $\boxed{i_1 \cong \frac{V_{Th}}{R} = \underset{\sim}{10 \cos(\omega t)}\ \underset{\sim}{mA}}$.

$\Rightarrow V_m = 10$, $I_m = 10 \times 10^{-3}$.

$$\Rightarrow P_{avg} = \frac{V_m I_m}{2} \underbrace{\cos(\theta_v - \theta_i)}_{=1} = \frac{V_m I_m}{2} = \frac{10 \times \overset{5}{\cancel{10}} \times 10^{-3}}{\cancel{2}} = 50\,mW.$$

$$p = P_{avg} + P_{avg} \cos(2\omega t) = [50 + 50 \cos(2\omega t)]\,mW.$$



# P43

Thursday, April 20, 2023 9:34 AM

## Purely Inductive Circuit:

If purely inductive $\Rightarrow \theta_i = \theta_v - 90°$ or $\boxed{\theta_v - \theta_i = 90°}$

$\Rightarrow$ Current lags voltage

and

$p = -Q\sin(2\omega t)$ (instantaneous power for purely inductive circuit)

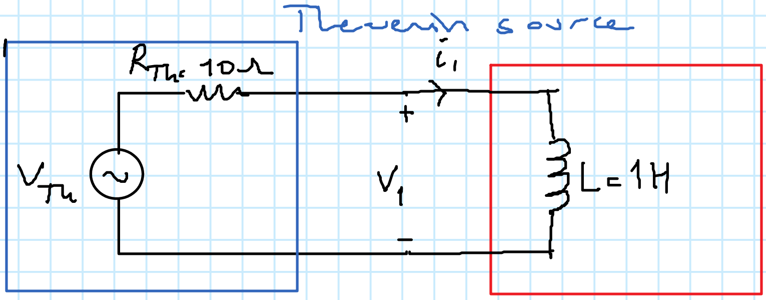
- The average power is zero. $P_{avg} = 0$, and we only have the reactive power Q.

To distinguish, for average power, we use the unit "watts", and for reactive power, we use the unit "volt-amp reactive" (VAR).

Ex:

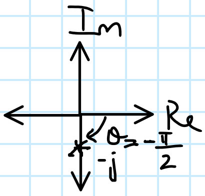
$V_{Th} = 10\cos(\omega t)$

$\omega = 2000\pi \left(\frac{rad}{sec}\right)$



Purely inductive circuit

Find Q and p.



Ans.

$V_1 \approx V_{Th}$ , $\bar{I}_1 \approx \frac{\bar{V}_{Th}}{Z} = \frac{10}{j\omega L} = \frac{10}{j(2000\pi)} = -j1.6\times10^{-3} = 1.6\,e^{-j\frac{\pi}{2}}$ mA $= 1.6\angle{-\pi/2}$ mA.

$\Rightarrow i_1(t) = \mathrm{Re}[\bar{I}_1 . e^{j\omega t}] = 1.6\cos(2000\pi t - \frac{\pi}{2})$ mA.

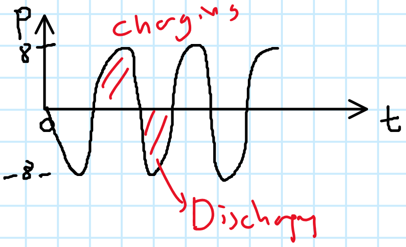
$\theta_i = -\frac{\pi}{2}$

$V_1(t) = 10\cos(2000\pi t)$ , $\theta_v = 0$ .

$\Rightarrow \theta_v - \theta_i = \frac{\pi}{2}$ (purely inductive)

$\Rightarrow Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{10(1.6\times10^{-3})}{2}\sin(\frac{\pi}{2}) = 8$ m VAR

and $p = -Q\sin(2\omega t) = -8\sin(4000\pi t)$ (mW)



# P44

Thursday, April 20, 2023 9:50 AM

## AC Power for Purely Capacitive Circuits:

In this case, $\theta_i = \theta_v + 90°$ or $\boxed{\theta_v - \theta_i = -90°}$ (current leads voltage)

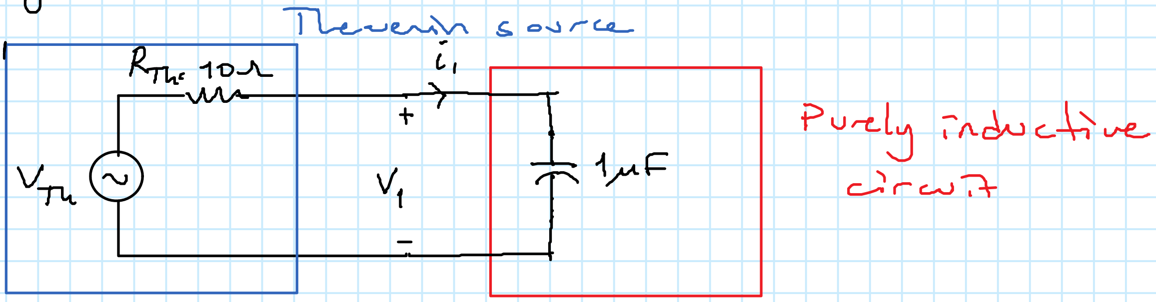
and $p = -Q\sin(2\omega t)$

$P_{avg} = 0$ W.

Ex:

$V_{Th} = 10\cos(\omega t)$

$\omega = 2000\pi \left(\frac{rad}{sec}\right)$.



Find Q and p.

Ans:

$$\bar{i}_1 \approx \frac{10}{\frac{-j}{\omega c}} = \frac{10}{\frac{-j}{(2000\pi)(10^{-6})}} = 63\text{mA}\, e^{j1.57}$$

$V_1 \approx V_{Th}$, To find $i_1(t)$:

$\Rightarrow i(t) = 63\cos(2000\pi t + \frac{\pi}{2})$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{10(63\times10^{-3})}{2}\sin(-\frac{\pi}{2}) = -315 \text{ mVAR}$$

$p = -Q\sin(2\omega t) = 315\sin(4000\pi t)$ mW. , $P_{avg} = 0$ W.

—0—

## Power Factor: ↓HW 8↓

- The angle $(\theta_v - \theta_i)$ = Power factor angle.
- The cosine of this angle is called "power factor"

i.e $\boxed{pf = \cos(\theta_v - \theta_i)}$.

For a purely resistive circuit, $pf = 1$ *

" " " inductive/capacitive, $pf = 0$

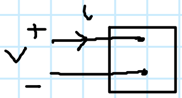
- For home appliances, we want pf to be as close to 1 as possible.

Ex:

a-) Calculate the average power and the reactive power at the terminals of the network shown in the figure below

$V = 100\cos(\omega t + 15°)$ V

$i = 4\sin(\omega t - 15°)$ A.



# P45

Thursday, April 27, 2023 9:45 AM

b-) State whether the network inside the box absorbs or delivers average power

c-) State whether the network inside the box absorbs or delivers VAR's

Ans:

a-) Using "cos" as the reference,

$$i = 4\sin(\omega t - 15°) = 4\cos(\omega t - 15° - 90°)$$

$$= 4\cos(\omega t - 105°) \text{ A.}$$

$$\Rightarrow \theta_v - \theta_i = 15° - (-105°) = 120°$$

$$\Rightarrow P_{avg} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{100 \cdot \overset{2}{\cancel{4}}}{\cancel{2}}\cos(120°) = -100\text{W.}$$

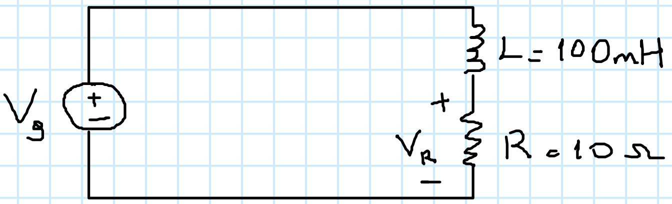
$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{100 \cdot 4}{2}\sin(120°) = 173.21 \text{ VAR's}$$

b-) Network inside the box delivers average power

c-) $Q > 0 \Rightarrow$ it is absorbing reactive power (inductive)

Ex.

For the following circuit, find the $P_{avg}$, $Q$.

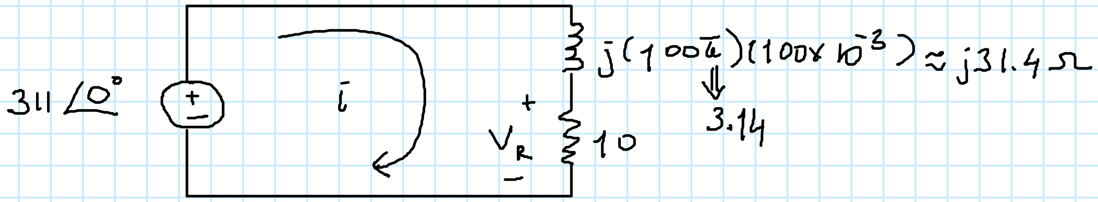


$V_g = 311\cos(\omega t)$, $\omega = 2\pi f = 2\pi(50) = 100\pi$ rad/sec.

Ans:

We can find the current and voltages in the circuit by using the phasor analysis.

The phasor domain circuit is given as:



# P46

Thursday, April 27, 2023 9:54 AM

$$\bar{i} = \frac{\bar{V}_g}{Z_{total}} = \frac{311\angle 0^\circ}{10+j31.4} = 2.864 - j9 \ A = 9.437\angle -1.26 = 9.437\angle -72.3^\circ A$$

$$\Rightarrow \theta = \theta_v - \theta_i = 0 - (-71^\circ) = 72.3^\circ \text{(inductive)}$$

$$pf = \cos(72.3^\circ) = 0.3$$

To find the real power:

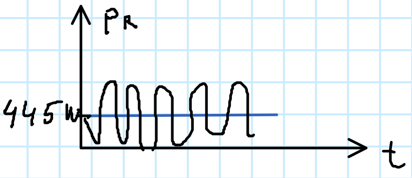
$$P_{avg} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = ?$$

$$\bar{V}_R = \bar{i}\cdot R = 94.37\angle -72.3^\circ \ V.$$

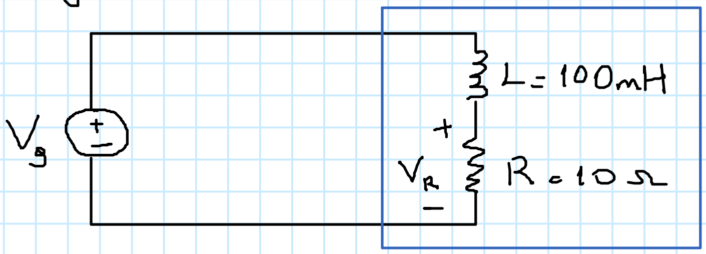
$$\Rightarrow P_{avg} = \frac{(94.37)(9.437)}{2} = 445 \ W$$

$$\downarrow$$

$$P_{avg} = \frac{V_m I_m}{2}\cos(\underbrace{\theta_v - \theta_i}_{\substack{\| \\ 0.}})$$



Alternatively.



$\bar{V}_g = 311\angle 0^\circ$, $\bar{i} = 9.43\angle -72^\circ$

$\theta = \theta_v - \theta_i = 72.3^\circ$

$pf = 0.3$

$$P_{avg} = \frac{(311)(9.43)}{2}(0.3)$$

$$\cong 445 W.$$

Also,

$$Q = \frac{V_m I_m}{2}\sin(\theta) = \frac{(311)(9.43)}{2}\sin(72.3^\circ) = \frac{(311)(9.43)}{2}(0.9527)$$

$$= 1397 \ VAR$$

$$\approx 1.4 \ kVAR.$$

P47

Thursday, April 27, 2023 10:17 AM

## - RMS Value and Power Calculations -

$V_{rms} = \frac{V_m}{\sqrt{2}}$ , $i_{rms} = \frac{I_m}{\sqrt{2}}$ , $P_{avg} = \frac{V_m I_m}{2}$ (for a resistor)

$P_{avg} = V_{rms} \cdot i_{rms}$

Also,

$$P_{avg} = \frac{V_m^2}{2R} = \frac{I_m^2}{2} R = \frac{V_{rms}^2}{R} = i_{rms}^2 \cdot R$$

Ex.

A sinusoidal voltage having a max amplitude of 625V is applied to the terminals of a 50Ω resistor. Find the average power delivered to the resistor?

Ans:

$V_m = 625V$ , $V_{rms} = \frac{V_m}{\sqrt{2}} \approx \frac{V_m}{1.4} = 441.54V$

$\Rightarrow P_{avg} = \frac{V_{rms}^2}{R} = \frac{(441.94)^2}{50} = 3.9 kW.$

Alternatively

$I_m = \frac{V_m}{R} = \frac{625}{50} = 12.5A.$ $\Rightarrow i_{rms} = \frac{I_m}{\sqrt{2}} = 8.84A$

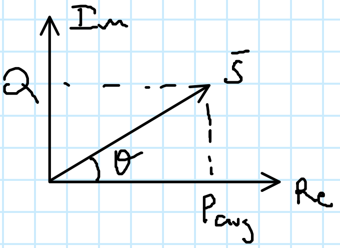
$\Rightarrow P_{avg} = i_{rms}^2 \cdot R = (8.84)^2 (50) = 3.9 kW.$

# - Complex Power -

Complex power is defined as

$$\boxed{S = P_{avg} + jQ}$$

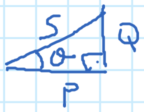
In the complex plane:



$\theta = \theta_v - \theta_i$.

$$\tan\theta = \frac{Q}{P_{avg}}$$

Power Triangle:



$$\boxed{|\bar{S}| = \text{Apparent power} = \sqrt{P_{avg}^2 + Q^2}}$$

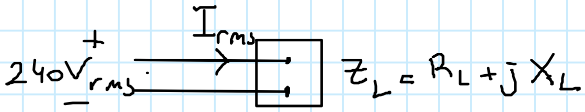
Ex:

An electrical load operates at 240 $V_{rms}$. The load absorbs an average power of 8kW at a $\underbrace{\text{lagging factor}}_{\text{lagging p.f.}}$ of 0.8

a-) calculate the complex power of the load.

b-) calculate the impedance of the load.

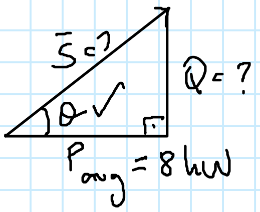
Ans:



p.f. = 0.8 (lagging) $\theta_v - \theta_i > 0$ (inductive)

$P_{avg} = 8\,kW$

a-) Let us draw the power triangle first.



$\theta = \theta_v - \theta_i = ?$

$pf = \cos(\theta_v - \theta_i) = \cos\theta = 0.8$

$\cos\theta = 0.8 \Rightarrow \sin\theta = 0.6$

From the power triangle.

$$P_{avg} = |\bar{S}| . \cos\theta$$

Also, $$Q = |\bar{S}| . \sin\theta$$

$$\Rightarrow |\bar{S}| = \frac{P_{avg}}{\cos\theta} = \frac{8000}{0.8} = 10 \text{ kVA.}$$

and $Q = 10 . \sin\theta$ kVAR $= 10 . 0.6$ kVAR's. $= 6$ kVARs.

$\Rightarrow \bar{S} = 8 \text{ kW} + j6 \text{ kVARs}$

b-) $Z_L = ?$ , $Z_L = \frac{V_{rms}\angle\theta_v}{I_{rms}\angle\theta_i} = \frac{V_{eff}\angle\theta_v}{I_{eff}\angle\theta_i}$

$$\underset{\underset{6\text{ kVARs.}}{\swarrow}}{Q} = \underset{\underset{240\text{ V}}{\downarrow}}{V_{eff}} . \underset{=}{I_{eff}} . \underset{\underset{0.6}{\downarrow}}{\sin\theta}$$

$\cos\theta = 0.8$

$\Rightarrow I_{eff} = 41.67$ A.

Thus,

$$Z_L = \frac{V_{eff}}{I_{eff}} \angle\theta_v - \theta_i = \frac{240}{41.67} \angle\theta = 5.76\angle 36.87^\circ$$

$$\Rightarrow \boxed{Z_L = 4.608 + j\,3456 \ \Omega .}$$

$R_L = 4.6 \ \Omega$

$\omega L = 3.456$ (Taking f=50 Hz.)

$\Rightarrow L = \frac{3.456}{100\pi} = 0.011 = 11$ mH.

-Power Calculations-

The complex power is $S = P + jQ$ (VA)

where

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

Then,

$$S = \frac{V_m I_m}{2} \left[ \cos\underbrace{(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)} \right]$$

$$= \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = \frac{V_m I_m}{2} \angle\theta_v - \theta_i = V_{eff} . I_{eff} . \angle\theta_v - \theta_i$$

Thus, $\boxed{S = V_{eff} I_{eff} \angle \theta_v - \theta_i}$, where $V_{eff} = V_{rms}$ and $I_{eff} = I_{rms}$.

Furthermore,

$$S = V_{eff} I_{eff} e^{j(\theta_v - \theta_i)} = \underbrace{V_{eff} e^{j\theta_v}}_{\bar{V}_{eff}} \cdot \underbrace{I_{eff} e^{-j\theta_i}}_{\bar{I}^*_{eff}} = \bar{V}_{eff} \cdot \bar{I}^*_{eff}$$

$$\Rightarrow \boxed{S = \bar{V}_{eff} \cdot \bar{I}^*_{eff}}$$

Alternatively,

$$\boxed{S = \frac{1}{2} \cdot \bar{V} \cdot \bar{I}^*}$$

Also,

$$\boxed{Z = \frac{\bar{V}_{eff}}{\bar{I}_{eff}}} \Rightarrow S = (Z \bar{I}_{eff}) \bar{I}^*_{eff} = |\bar{I}_{eff}|^2 Z \quad (VAR)$$

$$I_{eff} e^{j\theta_i} \cdot I_{eff} e^{-j\theta_i}$$

or

$$\boxed{S = |\bar{I}_{eff}|^2 Z = |\bar{I}_{eff}|^2 (R + jX)} = \underbrace{|I_{eff}|^2 R}_{P_{av}} + j \underbrace{|I_{eff}|^2 X}_{Q}$$

or

$$\boxed{S = \frac{1}{2} |\bar{I}|^2 Z}$$

Similarly,

$$\boxed{S = \frac{|V_{eff}|^2}{Z^*}} = P_{av} + jQ \quad , \text{where } P_{av} = \frac{|V_{eff}|^2}{R}$$

or

$$\boxed{S = \frac{|\bar{V}|^2}{2Z^*}} \qquad \text{and } Q = \frac{|V_{eff}|^2}{X}$$

**Ex**

Given $\bar{V} = 100 \angle 15^\circ$ (V) and $\bar{I} = 4 \angle -105^\circ$ (A) ⇒ Find S.

**Ans**

Using

$$S = \frac{1}{2} \cdot \bar{V} \cdot \bar{I}^* = \frac{1}{2} (100 \angle 15^\circ)(4 \angle -105^\circ)^* = 200 \angle 15^\circ - (-105)^\circ$$

$$= 200 \angle 120^\circ$$

$$= -100 + j173\ 21 \ (VA)$$

Then, $P_{av} = -100W$ (source)

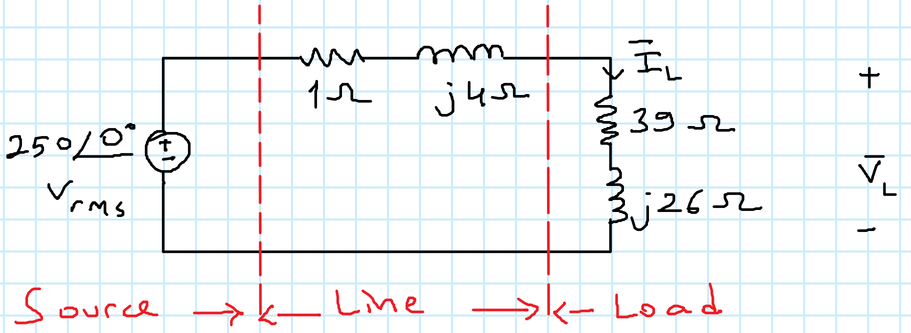
and $Q = 173.21$ VARs.

# P51

Monday, April 5, 2021 11:35 AM

**Ex:**

Given the circuit:



Calculate

a-) The load current $\bar{I}_L$ and voltage $\bar{V}_L$

b-) The average and reactive power delivered to the load.

c-) The " " " " " " " Line

d-) " " " " " supplied by the source.

**Ans**

a-) $\bar{I}_L = \frac{\bar{V}_{source}}{Z_{total}} = \frac{250\angle 0^\circ}{40+j30} = 4-j3$ (A) $= 5\angle -36.87^\circ$ (A)(rms) $= \bar{I}_{eff}$

$$\bar{V}_L = \bar{I}_L \cdot Z_L = (5\angle -36.87^\circ)\underbrace{(39+j26)}_{\substack{=46.87\angle 0.588 \\ =46.87\angle 33.7^\circ}} = 234.36\angle -3.18^\circ \text{ (V)}_{rms}$$

b-) $S = \bar{V}_{eff} \cdot \bar{I}^*_{eff} \Rightarrow S_L = \bar{V}_{L_{eff}} \cdot \bar{I}^*_{L_{eff}} = (234.36\angle -3.18^\circ)(5\angle 36.87^\circ)$

$= 975 + j650$ VA

Thus, the load absorbs average power of 975 W and reactive power of 650 VArs.

c-) $\bar{P}_{avg_{line}} = |I_{eff}|^2 R_{line} = (5)^2(1) = 25$ W.

$Q = |I_{eff}|^2 X = (5)^2(4) = 100$ VARs.

# P52

Thursday, April 27, 2023 10:59 AM

d-) $S = \underbrace{(25 + j100)}_{S_{Line}} + \underbrace{(975 + j650)}_{S_L} \Rightarrow S_{source} = 1000 + j750$ VA . is supplied.

or

$S_{source} = -1000 - j750$ VA.

## — HW 9 —

## Introduction to the Laplace Transform —

The Laplace transform is defined as

$$\boxed{\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st}\,dt}$$

(one-sided unilateral Laplace Transform)

The operator $\mathcal{L}\{f(t)\}$ = Laplace transform of $f(t)$

Also,

$F(s) = \mathcal{L}\{f(t)\}$ where $\mathcal{L}$ = Laplace operator

$s$ = Laplace variable. $= j\omega$

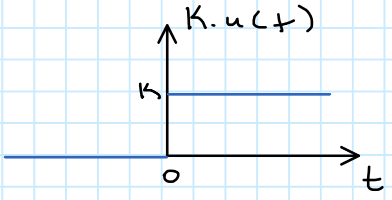
Some important functions are

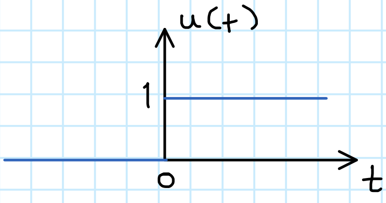
→ Step Function:

It is defined as,

$$\boxed{K.u(t) = \begin{cases} K.u(t) = 0, & t < 0 \\ K.u(t) = 1, & t > 0 \end{cases}}$$

If K=1, then the step function is called "unit step function". (u(t)).



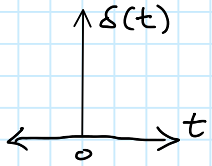


- It is not defined at $t=0$.

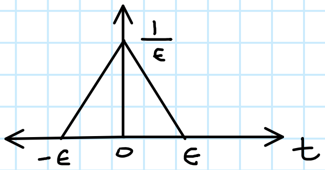
# P53

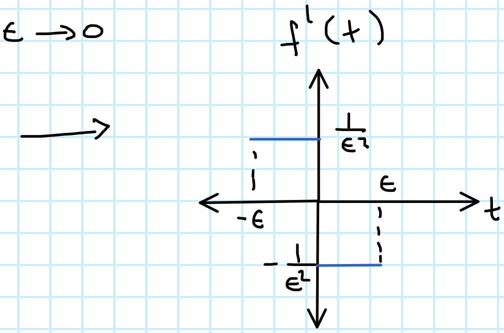
Thursday, May 4, 2023 8:00 AM

- Impulse Function $= \delta(t)$



$f(t) \approx \delta(t)$ as $\epsilon \to 0$





# P54

Thursday, May 4, 2023 8:04 AM

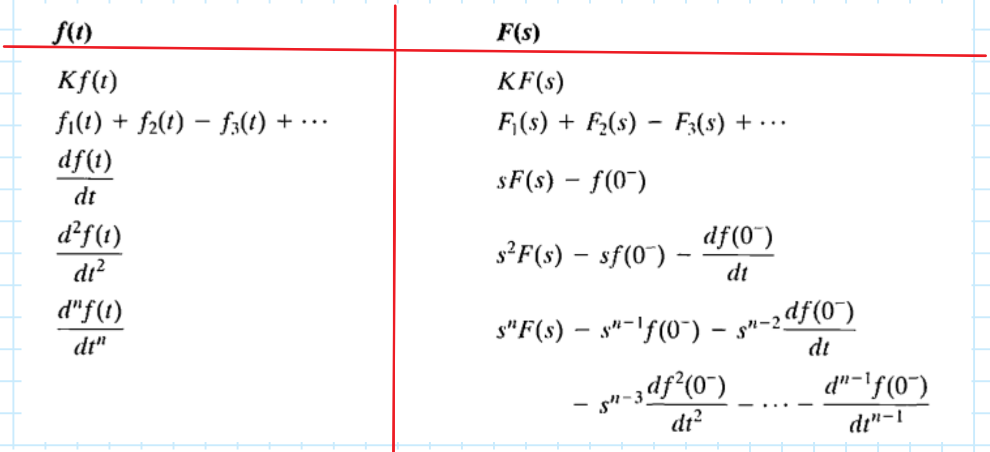
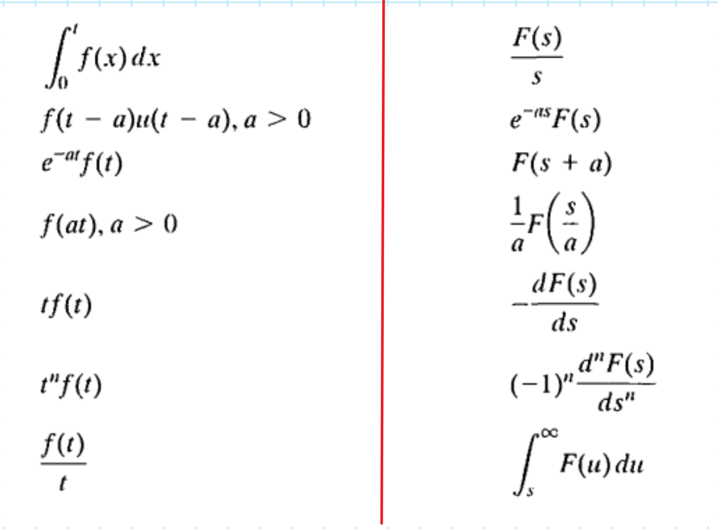
## Functional Transforms:

$$\mathcal{L}\{u(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt = \int_0^\infty 1 .\, e^{-st}\, dt = \frac{e^{-st}}{-s}\Big|_0^\infty = 1.$$

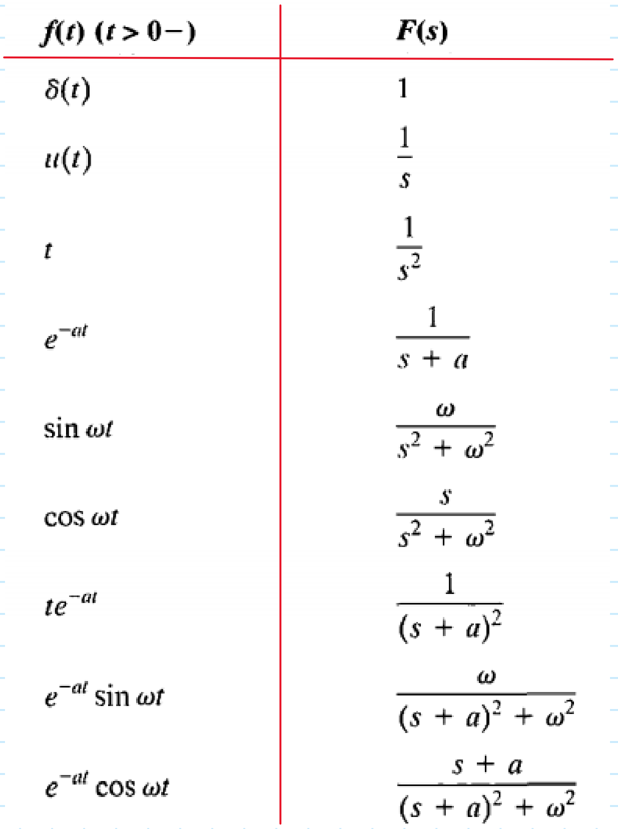
$$\mathcal{L}\{e^{-at}\} = \int_0^\infty e^{-at} \cdot e^{-st}\, dt = \int_0^\infty e^{-(a+s)t}\, dt = \frac{1}{s+a}$$

$$\mathcal{L}\{\sin \omega t\} = \int_0^\infty \sin \omega t \; e^{-st}\, dt = \int_0^\infty \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \cdot e^{-st} \cdot dt = \frac{\omega}{s^2+\omega^2}$$

## List of Laplace Transform Pairs.

| $f(t)$ | $F(s)$ |
|---|---|
| $Kf(t)$ | $KF(s)$ |
| $f_1(t) + f_2(t) - f_3(t) + \cdots$ | $F_1(s) + F_2(s) - F_3(s) + \cdots$ |
| $\frac{df(t)}{dt}$ | $sF(s) - f(0^-)$ |
| $\frac{d^2f(t)}{dt^2}$ | $s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$ |
| $\frac{d^nf(t)}{dt^n}$ | $s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$ |
| $\int_0^t f(x)\,dx$ | $\frac{F(s)}{s}$ |
| $f(t-a)u(t-a), a > 0$ | $e^{-as}F(s)$ |
| $e^{-at}f(t)$ | $F(s+a)$ |
| $f(at), a > 0$ | $\frac{1}{a}F\left(\frac{s}{a}\right)$ |
| $tf(t)$ | $-\frac{dF(s)}{ds}$ |
| $t^nf(t)$ | $(-1)^n\frac{d^nF(s)}{ds^n}$ |
| $\frac{f(t)}{t}$ | $\int_s^\infty F(u)\,du$ |



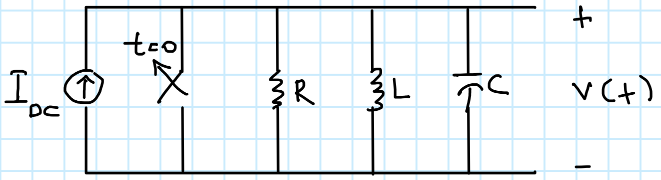
| $f(t)\ (t > 0-)$ | $F(s)$ |
|---|---|
| $\delta(t)$ | 1 |
| $u(t)$ | $\frac{1}{s}$ |
| $t$ | $\frac{1}{s^2}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ |
| $te^{-at}$ | $\frac{1}{(s+a)^2}$ |
| $e^{-at}\sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ |
| $e^{-at}\cos \omega t$ | $\frac{s+a}{(s+a)^2+\omega^2}$ |

# P55

Thursday, May 4, 2023 9:45 AM

## Applying the Laplace Transform:

Consider the following circuit.



Given that the initial energy stored in the circuit is zero Find v(t) for $t \geq 0$.

Thus, this is a transient response problem of a parallel RLC circuit

Now, let us see how we can solve this problem by the Laplace transform

Node-voltage equation is.

$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(x)\,dx + C\frac{dv(t)}{dt} = I_{DC}\,u(t)$$

- Take the Laplace transform of both sides

$$\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C\left[sV(s) + v(0^+)\right] = I_{DC}\cdot\frac{1}{s}$$

where V(s) is unknown and R, L, C, $v(0^+)$ and $I_{DC}$ are known.

Then,

$$V(s)\left(\frac{1}{R} + \frac{1}{sL} + sC\right) = \frac{I_{DC}}{s}$$

or

$$V(s) = \frac{I_{DC}/C}{s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC}}$$

In order to find v(t), we need to find the "inverse transform" of V(s), i.e.

$$\boxed{v(t) = \mathcal{L}^{-1}\{V(s)\}}$$

## - Inverse Laplace Transform :

In general, we need to find the inverse Laplace transform of a function which has the following form:

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0} \quad , m, n = \text{integers.}$$

If $m > n \Rightarrow F(s)$ : proper

If $m \leq n \Rightarrow F(s)$ : improper

## Partial Fraction Method for the Solution of proper F(s) :

Let us consider the following example

$$F(s) = \frac{s+6}{s(s+3)(s+1)^2}$$

Then, $D(s)$ has 4 roots : $s=0$ , $s=-3$ , $s=-1$ (two roots at $s=-1$)

The partial fraction form is

$$\frac{s+6}{s(s+3)(s+1)^2} = \frac{K_1}{s} + \frac{K_2}{s+3} + \frac{K_3}{(s+1)^2} + \frac{K_4}{(s+1)}$$

Then,

$$\mathcal{L}^{-1}\left\{\frac{s+6}{s(s+3)(s+1)^2}\right\} = \left[K_1 + K_2 e^{-3t} + K_3 \cdot t e^{-t} + K_4 e^{-t}\right] u(t)$$

We need to find the coefficients $K_1, K_2, K_3$ and $K_4$.

There are 4 cases of this problem.

The roots of $D(s)$ are either

1-) Real and distinct.

2-) Complex and distinct

3-) Real and repeated.

4-) Complex and repeated

## 1-) Real and Distinct Roots of $D(s)$:

Assume we have

$$F(s) = \frac{96\,(s+5)(s+12)}{s\,(s+8)(s+6)} = \frac{K_1}{s} + \frac{K_2}{s+8} + \frac{K_3}{s+6}$$

To find the value of $K_1$, we multiply both sides by $s$, and evaluate the both sides at $s=0$

$$\left.\frac{96\,(s+5)(s+12)}{(s+8)(s+6)}\right|_{s=0} = K_1 + \left.\cancelto{0}{\frac{K_2 s}{s+8}}\right|_{s=0} + \left.\cancelto{0}{\frac{K_3 s}{s+6}}\right|_{s=0}$$

$$\frac{96\cdot 5\cdot 12}{8\cdot 6} = K_1 \;\Rightarrow\; K_1 = 120$$

To find $K_2$, multiply both sides by $(s+8)$ and evaluate both sides at $s=-8$.

$\vdots$

$$K_2 = -72, \quad K_3 = 48.$$

Then,

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = [120 - 72e^{-8t} + 48e^{-6t}]\,u(t)$$

## 2-) Complex and Distinct Roots of $D(s)$:

Consider

$$F(s) = \frac{100(s+3)}{(s+6)\underbrace{(s^2+6s+25)}_{(s+3-j4)(s+3+j4)}}$$

$$\Rightarrow \quad F(s) = \frac{K_1}{s+6} + \frac{K_2}{s+3-j4} + \frac{K_3}{s+3+j4}$$

Then,

$$K_1 = \left.\frac{100\,(s+3)}{(s^2+6s+25)}\right|_{s=-6} = \frac{100\,(-3)}{25} = -12$$

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and

$$K_2 = \frac{100(s+3)}{(s+6)}\Bigg|_{s=-3+j4} = 6 - j8 = 10e^{-j53^\circ}$$

and

$$K_3 = \frac{100(s+3)}{(s+6)}\Bigg|_{s=-3-j4} = 6 + j8 = 10e^{j53^\circ}$$

Then,

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \left[-12e^{-6t} + 10e^{-j53^\circ} e^{-(3-j4)t} + 10e^{j53^\circ} e^{-(3+j4)t}\right]u(t)$$

Using the Identity

$$\mathcal{L}^{-1}\left\{\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}\right\} = 2|K|e^{-\alpha t}\cos(\beta t+\theta)$$

$$\Rightarrow \mathcal{L}^{-1}\{F(s)\} = \left[-12e^{-6t} + 20e^{-3t}\cos(4t+53^\circ)\right]u(t)$$

—o—

## 3-) Real and Repeated Roots of D(s)

Consider

$$F(s) = \frac{100(s+25)}{s(s+5)^3} = \frac{K_1}{s} + \frac{K_2}{(s+5)^3} + \frac{K_3}{(s+5)^2} + \frac{K_4}{(s+5)}$$

and

$$K_1 = \left.\frac{100(s+25)}{(s+5)^3}\right|_{s=0} = \frac{100(25)}{125} = 20.$$

- To find $K_2$, multiply both sides by $(s+5)^3$, and evaluate at $s=-5$.

$$\Rightarrow K_2 = \frac{100(20)}{-5} = -400$$

- To find $K_3$, multiply both sides by $(s+5)^3$, and then differentiate once with respect to $s$, and evaluate at $s=-5$.

$$\frac{d}{ds}\left[\frac{100(s+25)}{s}\right]_{s=-5} = \frac{d}{ds}\left[\frac{K_1(s+5)^3}{s}\right]_{s=-5} + \frac{d}{ds}\left[K_2\right]_{s=-5}$$

$$+ \frac{d}{ds}\left[K_3(s+5)\right]_{s=-5} + \frac{d}{ds}\left[K_4(s+5)^2\right]_{s=-5}$$

$$\Rightarrow K_3 = 100\left[\frac{s-(s+25)}{s^2}\right]_{s=-5} = -100$$

-> To find $K_4$, multiply both sides by $(s+5)^3$, and differentiate twice w.r.t $s$, and evaluate at $s=-5$

$$\vdots$$

$$\Rightarrow K_4 = -20.$$

$$\Rightarrow \mathcal{L}^{-1}\{F(s)\} = \left[20 - 200t^2e^{-5t} - 100te^{-5t} - 20e^{-5t}\right]u(t)$$

## 4) Complex and Repeated Roots

Consider

$$F(s) = \frac{768}{\underbrace{(s^2+6s+25)^2}_{(s+3-j4)(s+3+j4)}}$$

$$F(s) = \frac{k_1}{(s+3-j4)^2} + \frac{K_2}{(s+3-j4)} + \frac{k_1^*}{(s+3+j4)^2} + \frac{K_2^*}{(s+3+j4)}$$

where

$$K_1 = \frac{768}{(s+3-j4)^2}\bigg|_{s=-3+j4} = \frac{768}{(j8)^2} = -12.$$

and

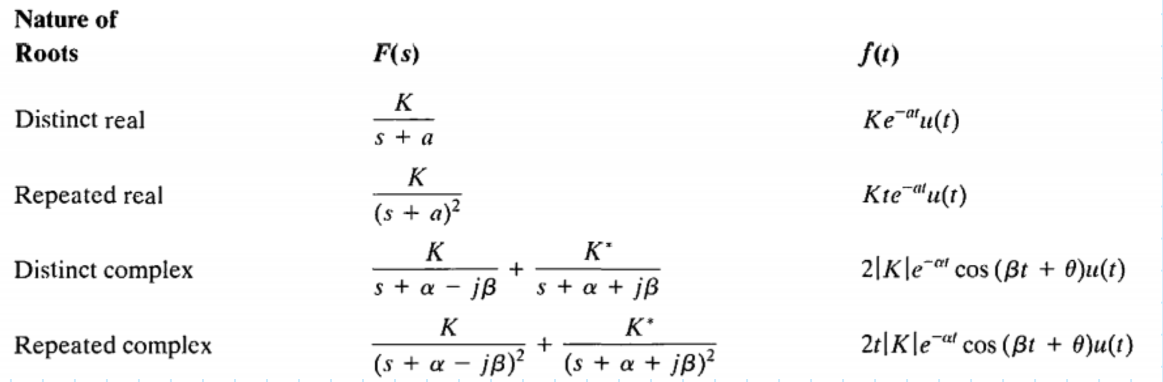
$$K_2 = \frac{d}{ds}\left[\frac{768}{(s+3+j4)^2}\right]\bigg|_{s=-3+j4} = \frac{2(768)}{(s+3+j4)^2}\bigg|_{s=-3+j4} = \frac{-2(768)}{(j8)^3}$$

$$\Rightarrow K_2 = -j3 = 3\angle -90^\circ$$

$K_1^* = -12$ , $K_2^* = j3 = 3\angle 90^\circ$

$$\Rightarrow f(t) = \mathcal{L}^{-1}\{F(s)\} = \left[-24te^{-3t}\cos(4t) + 6e^{-3t}\cos(4t-90^\circ)\right]u(t).$$

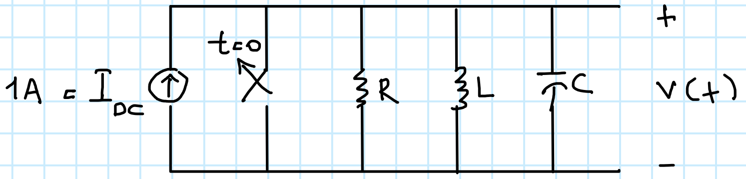
In summary, the compact formulations for all 4 cases are given below:



| Nature of Roots | $F(s)$ | $f(t)$ |
|---|---|---|
| Distinct real | $\frac{K}{s+a}$ | $Ke^{-at}u(t)$ |
| Repeated real | $\frac{K}{(s+a)^2}$ | $Kte^{-at}u(t)$ |
| Distinct complex | $\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}$ | $2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$ |
| Repeated complex | $\frac{K}{(s+\alpha-j\beta)^2} + \frac{K^*}{(s+\alpha+j\beta)^2}$ | $2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$ |

* * *

Ex



$R = 1k\Omega$, $L = 1mH$, $C = 10\mu F$.

$$V(s) = \frac{I_{DC}/C}{s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC}} = \frac{1/10\times10^{-6}}{s^2 + \left(\frac{1}{10\times10^{-3}}\right)s + \frac{1}{10\times10^{-9}}}$$

or

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$$V(s) = \frac{10^5}{\underbrace{s^2+100s+10^8}_{(s+50-10000j)(s+50+10000j)}}$$

$$\Rightarrow \quad V(s) = \frac{K_1}{s+\underset{\alpha}{50}-\underset{\beta}{10000j}} + \frac{K_2}{s+50+10000j}$$

To find $K_1$:

Multiply both sides by $(s^2+100s+10^8)$ and evaluate at $s=-50+10000j$

$$10^5 = K_1(s+50+10000j)\Big|_{s=-50+10000j} + K_2\cancelto{0}{(s+50-10000j)}\Big|_{s=-50+10000j}$$

$$\Rightarrow K_1 = \frac{10^5}{20000j} = -5j = 5\angle{-90^\circ}$$

$$K_2 = 5\angle{90^\circ}$$

$$v(t) = \mathcal{L}^{-1}\{V(s)\} = 2|K|e^{-\alpha t}\cos(\beta t+\theta)$$

$$= 10\,e^{-50t}\cos\left(10000t-\frac{\pi}{2}\right)u(t)\ \text{V.}$$

* * *

## Partial Fraction Expansion for Improper Rational - Functions -

Consider

$$F(s) = \frac{s^4+13s^3+66s^2+200s+300}{s^2+9s+20}$$

After polynomial division

$$\begin{array}{l|l}
s^4+13s^3+66s^2+200s+300 & s^2+9s+20 \\ \hline
-(s^4+9s^3+20s^2) & s^2+4s+10 \\ \hline
\quad 4s^3+46s^2+200s+300 & \\
-(4s^3+36s^2+80s) & \\ \hline
\quad 10s^2+120s+300 & \\
-(10s^2+90s+200) & \\ \hline
\quad 30s+100 &
\end{array}$$

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$$\Rightarrow F(s) = s^2+4s+10 + \underbrace{\frac{30s+100}{s^2+9s+20}}_{} \xrightarrow{} \text{Remainder}$$

$$= \frac{30s+100}{(s+4)(s+5)} = \frac{-20}{s+4} + \frac{50}{s+5}$$

Then,

$$f(t) = \frac{d^2\delta(t)}{dt^2} + 4\frac{d\delta(t)}{dt} + 10\delta(t) - \left(20e^{-4t} - 50e^{-5t}\right)u(t)$$

## Poles and Zeros of F(s)

Consider

$$F(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_n)}{(s+p_1)(s+p_2)\cdots(s+p_m)}$$

- The roots of the denominator polynomial $-p_1, -p_2, \ldots, p_m$ are called "poles of F(s)"
- The roots of the numerator polynomial $-z_1, -z_2, \cdot, -z_n$ are called "zeros of F(s)"

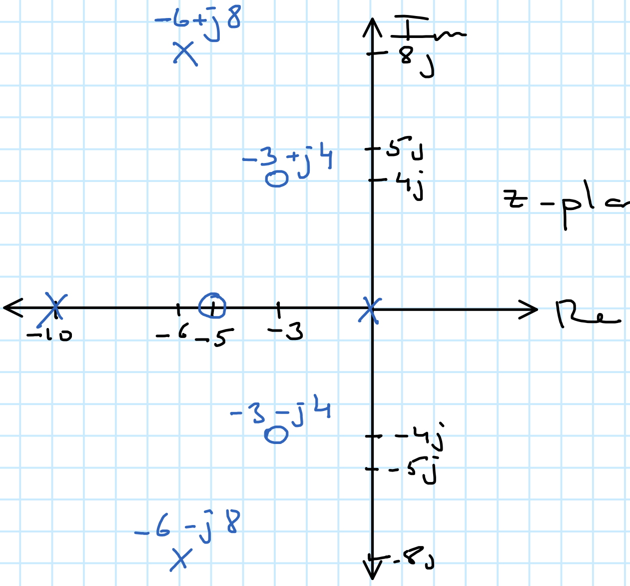
Ex

Given

$$F(s) = \frac{10(s+5)(s+3-j4)(s+3+j4)}{s(s+10)(s+6-j8)(s+6+j8)}$$

Show the zeros and poles on the z-plane.

Ans:



## – Laplace Transform in Circuit Analysis –

### Circuit Elements in the s-domain:

**Resistor**

$v = Ri$ (Ohm's law)

Since R is constant, the Laplace transform of Ohm's law is $V = RI$ where $V = \mathcal{L}\{v\}$, $I = \mathcal{L}\{i\}$, $\mathcal{L}\{R\} = R$

**Inductor**

$v = L \cdot \frac{di}{dt}$ is given. Take the Laplace transform of both sides,

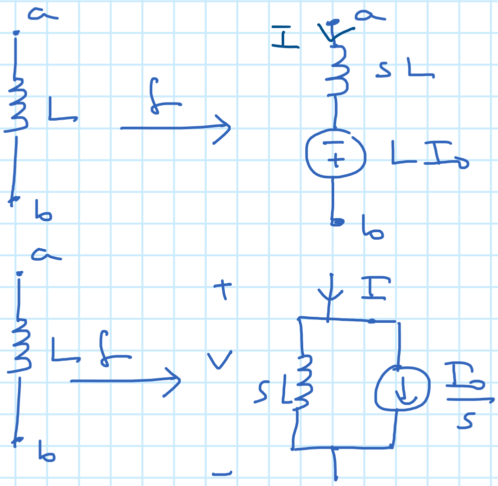
$$V = L[sI - i(0^-)] = sLI - LI_0$$

or

$$I = \frac{V + LI_0}{sL} = \frac{V}{sL} + \frac{I_0}{s}$$

If $I_0$ = Initial current = 0, then

$V = sLI$ where $\boxed{Z_L = sL}$ $(\Omega)$



**Capacitor**

$i = C\frac{dv}{dt}$ is given Take the Laplace transform of both sides,

$$I = C[sV - v(0^-)]$$

or

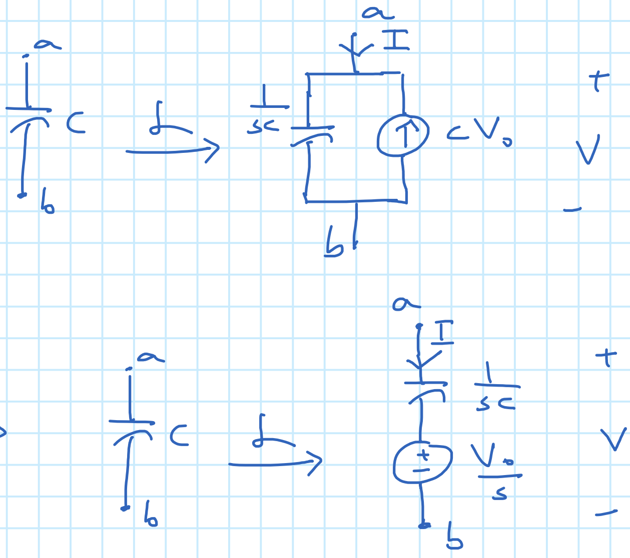
$$I = sCV - CV_0$$

or

$$V = \left(\frac{1}{sC}\right)I + \frac{V_0}{s}$$

If $V_0 = 0$, then,

$V = \left(\frac{1}{sC}\right)I \Rightarrow \boxed{Z_s = \frac{1}{sC}}$ $(\Omega)$



### Circuit Analysis in the s-domain:

$V = ZI$ (in the s-domain)

$\Sigma I = 0$ ) ,

$\Sigma V = 0$ $\Big\}$ Kirchoff's laws

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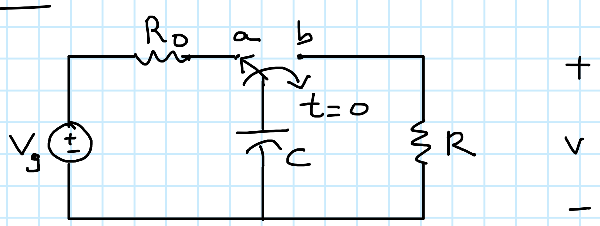
Ex:



Find v for $t \geq 0$ using Laplace transform

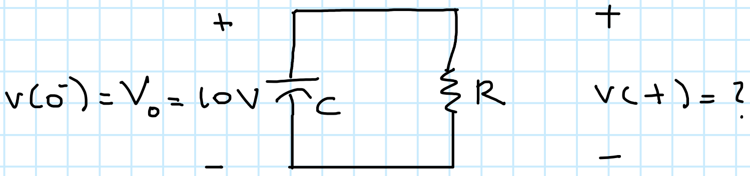
Given

$R_o = 100\Omega$, $C = 10\mu F$, $R = 1k\Omega$.

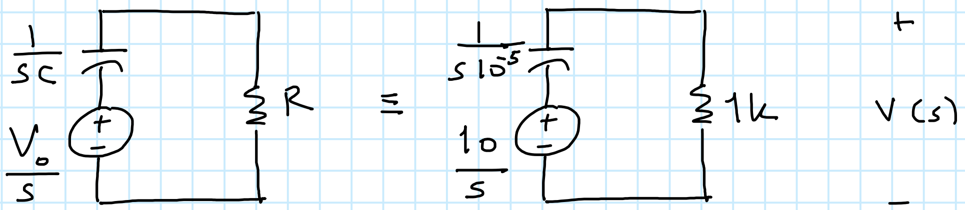
Also, $V_g = 10V$. (DC)

Ans:

- We have the natural response of an RC circuit.
- Let us find the initial capacitor voltage $V_0 = V_g = 10V$.
- The circuit for $t \geq 0$ is



- We convert the circuit into s-domain.



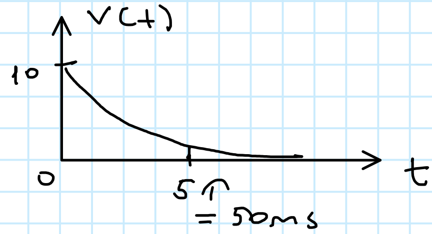
Using the voltage division

$$v(s) = \frac{10}{s} \cdot \frac{1k}{1k + \frac{10^5}{s}} = \frac{10}{\cancel{s}} \cdot \frac{1000\cancel{s}}{1000s + 10^5} = \frac{10000}{1000s + 10^5}$$

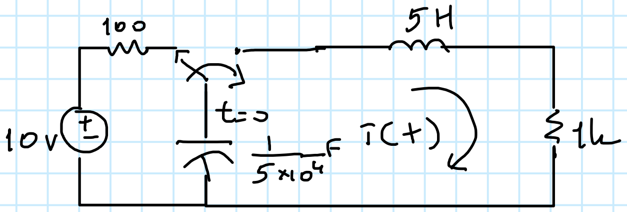
$$= \frac{10}{s + 100}$$

$$\Rightarrow v(t) = \mathcal{L}^{-1}\{v(s)\} = 10e^{-100t}u(t)$$

$$\tau = \frac{1}{100} = 10ms$$



Ex

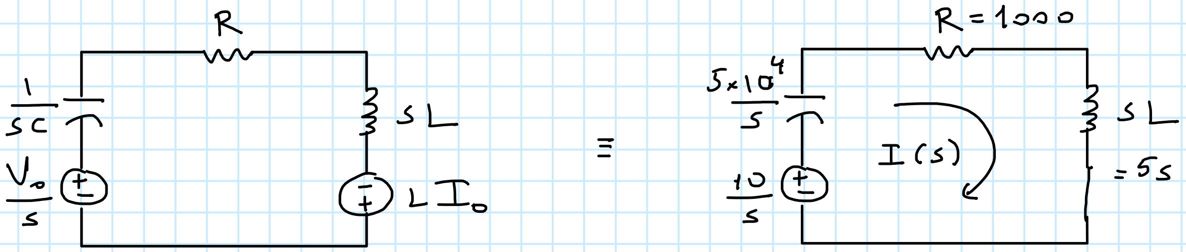


Find i(t) for t ≥ 0 using the Laplace transform

Ans:

- This is a natural response of a series RLC circuit.
- Initial conditions are $V_0$ = Initial capacitor voltage = 10 V.

  $I_0$ = Initial inductor current = 0.
- The circuit in s-domain:



$$\Rightarrow I(s) = \frac{10/s}{\underset{(s)}{\frac{1000}{1}} + \underset{(s)}{\frac{5s}{1}} + \underset{(1)}{\frac{5\times10^4}{s}}} = \frac{10}{\cancel{s}} \cdot \frac{\cancel{s}}{5s^2+1000s+5\times10^4}$$

$$= 10 \cdot \frac{1}{5s^2+1000s+5\times10^4} \frac{(1/5)}{(1/5)}$$

$$= 2 \cdot \frac{1}{s^2+200s+10^4}$$

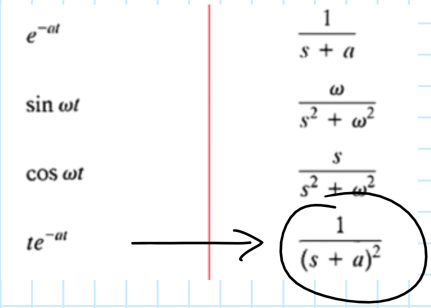
$$= \frac{k_1}{(s+100)^2} + \frac{k_2}{s+100}$$

⇒ Find $k_1$ and $k_2$ and find $\mathcal{L}\{I(s)\}$.

$$\frac{2}{s^2+200s+10^4} = \frac{k_1}{(s+100)^2} + \frac{k_2}{s+100}$$

$$\Rightarrow k_1 = 2,\ k_2 = 0.$$

$$i(t) = 2te^{-100t}u(t)$$



| | |
|---|---|
| $e^{-at}$ | $\frac{1}{s+a}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ |
| $te^{-at}$ → | $\frac{1}{(s+a)^2}$ |

# P66

Monday, May 24, 2021 9:58 AM

## - Transfer Function -

The transfer function is the s-domain ratio of the output (response) to the input (source)

If a circuit has multiple independent sources, we can find the transfer function for each source and use superposition (sum) to find the overall response to all sources

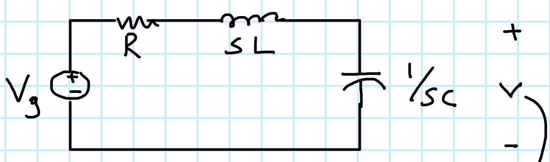
The transfer function is defined as

$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

where $Y(s)$. Laplace transform of the output signal.

$X(s)$ Laplace transform of the input signal

For example,



If the current v is taken as the response of the circuit and $V_g$ is the source, then:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{V(s)}{V_g(s)} \Rightarrow$$

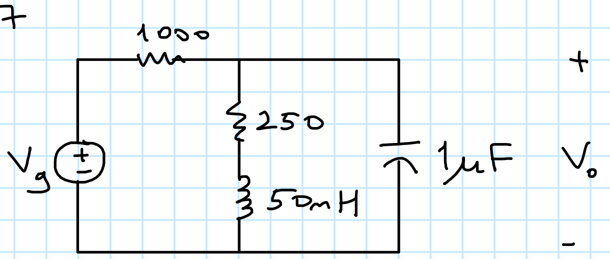
where

$$V(s) = V_g(s) \underbrace{\frac{1/sC}{\frac{1}{sC} + sL + R}}_{H(s)}$$

$$\Rightarrow H(s) = \frac{1}{s^2LC + sRC + 1}$$

Ex:

Given the circuit



$V_g = 50t\,u(t)$

Find H(s).

Find $V_o(t)$ using H(s).

# P67

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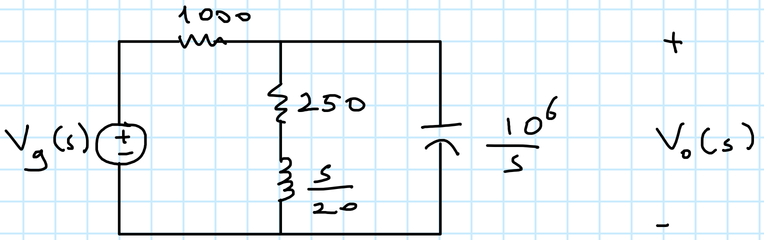
Ans: s-domain circuit



$$H(s) = \frac{V_o(s)}{V_g(s)} \Rightarrow$$ Using the voltage division

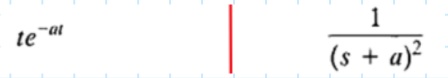
$$V_o(s) = V_g(s) \underbrace{\frac{\left(\frac{10^6}{s}\right) \parallel \left(250 + \frac{s}{20}\right)}{1000 + \left[\frac{10^6}{s} \parallel \left(250 + \frac{s}{20}\right)\right]}}_{H(s)}$$

⋮

$$\Rightarrow H(s) = \frac{1000\,(s+5000)}{s^2 + 6000\,s + 25\times10^6}$$

$$\Rightarrow V_o(s) = X(s) \cdot H(s) = \frac{50}{s^2} \cdot \frac{1000\,(s+5000)}{s^2 + 6000\,s + 25\times10^6}$$

where $X(s)$:



| f(t) | f(s) |
|---|---|
| $te^{-at}$ | $\frac{1}{(s+a)^2}$ |

$x(t) = 50\,t\,u(t)$ $\qquad \frac{50}{(s+0)^2} = \frac{50}{s^2}$

— — — — —

⋮

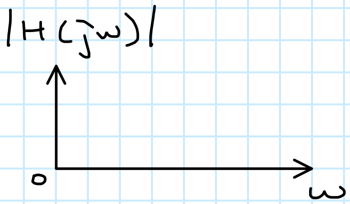
$$V_o(s) = \frac{5\sqrt{5}\times10^{-4}\,\angle{-79.7^\circ}}{s + 3000 + j4000} + \frac{10}{s^2} - \frac{4\times10^{-4}}{s}$$

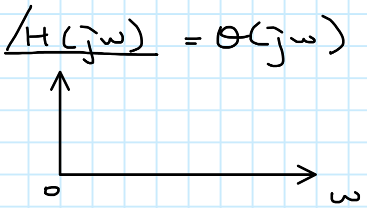
$$\Rightarrow v_o(t) = \left[10\sqrt{5}\times10^{-4}\,e^{-3000t}\cos(4000t + 79.7^\circ) + 10t - 4\times10^{-4}\right]u(t)$$

—o—

## Frequency Selective Circuits (Filters):

- Frequency selective circuit means that when the input frequency changes the output changes
- Filter analysis is done by using the transfer function $H(s)$
- We replace the variable $s = j\omega$, and obtain $H(j\omega)$ as a complex expression.
- Then, we plot



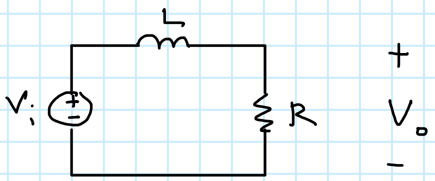


- These 2 plots are called the "Bode Plots"

There are 4 types of frequency response characteristics:

## 1-) Low-Pass Filter (LPF)

- This circuit passes low frequencies, and blocks (filters out) high frequencies
- We can implement a LPF from a serial RL circuit as:



The transfer function is:

$$H(s) = \frac{R}{R + sL} \quad \frac{(1/L)}{(1/L)}$$

$$\Rightarrow H(s) = \frac{R/L}{s + R/L}$$ (Transfer func. in s-domain)

Now, replace $\boxed{s = j\omega.}$

$$H(j\omega) = \frac{R/L}{j\omega + \frac{R}{L}}$$ (Transfer function in frequency domain)

# P69

Monday, May 24, 2021 10:58 AM

Let us find

$$|H(j\omega)| = \frac{|R/L|}{|j\omega + \frac{R}{L}|}$$

$$\frac{R}{L} + j\omega = a + jb$$

$$\Rightarrow |a+jb| = \sqrt{a^2 + b^2}$$

$$\Rightarrow \boxed{|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}}$$

and

$$\theta(j\omega) = \angle H(j\omega) = ?$$

where

$$H(j\omega) = \frac{R/L}{j\omega + \frac{R}{L}}.$$

$$\frac{a+j}{c+jd} = \frac{|a+jb|.e^{j\tan^{-1}\left(\frac{b}{a}\right)}}{|c+jd|.e^{j\tan^{-1}\left(\frac{d}{c}\right)}}$$

$|H(j\omega)|$

$$= |H(j\omega)|.e^{j\left[\underbrace{\tan^{-1}\frac{b}{a} - \tan^{-1}\left(\frac{d}{c}\right)}_{\angle H(j\omega) = \theta(j\omega)}\right]}$$

Thus, for this circuit (LPF)

$$\angle H(j\omega) = \theta(j\omega) = \cancelto{0}{\tan^{-1}\left(\frac{0}{\frac{R}{L}}\right)} - \tan^{-1}\left(\frac{\omega}{R/L}\right)$$

$$\Rightarrow \boxed{\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)}$$

—o—

# P70

Monday, May 24, 2021 11:33 AM

Thus, we have the following Bode plots

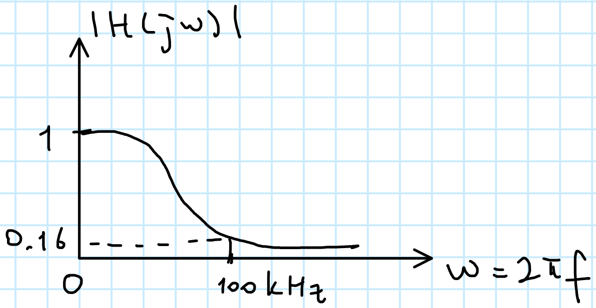
$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2+\left(\frac{R}{L}\right)^2}}$$

At $\omega = 0$

$|H(j\omega)| = 1$

At $\omega = \infty$

$H(j\omega) = 0$



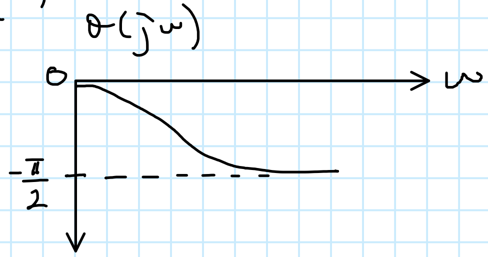
For $\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

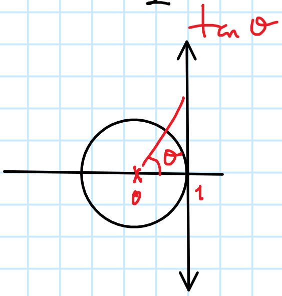
At $\omega = 0$

$\theta(j\omega) = 0$

At $\omega = \infty$

$\theta(j\omega) = -\frac{\pi}{2}$





# P71

Monday, May 31, 2021 10:24 AM

→ Half power frequency

## Cut-off Frequency (Corner Frequency or -3dB frequency).

- It is defined as the frequency when $|H(j\omega)| = \frac{1}{\sqrt{2}} = 0.707$.

Ex.

Find the cut-off frequency of the series RL-LPF.

Ans:

$$\boxed{|H(j\omega)| = \frac{R/L}{\sqrt{\omega_c^2 + \left(\frac{R}{L}\right)^2}} = \frac{1}{\sqrt{2}}}$$

Take the square of both sides,

$$\frac{R^2/L^2}{\omega_c^2 + \left(\frac{R}{L}\right)^2} = \frac{1}{2} \Rightarrow 2\left(\cancel{\frac{R}{L}}\right)^{2} = \omega_c^2 + \left(\cancel{\frac{R}{L}}\right)^{2}$$

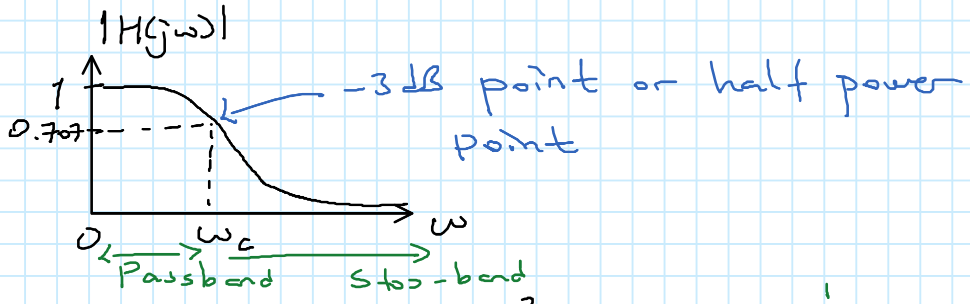
$$\omega_c^2 = \left(\frac{R}{L}\right)^2$$

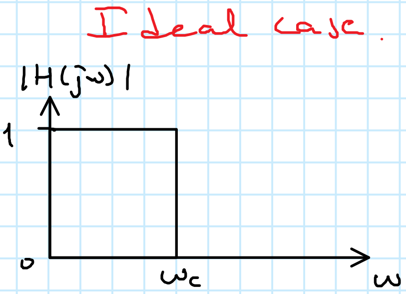
$$\Rightarrow \omega_c = \frac{R}{L} \quad \text{or} \quad f_c = \frac{1}{2\pi}\frac{R}{L} \ (Hz)$$

—o—

- Why do we equate to $\sqrt{2}$ ? or why is the corner frequency is called -3dB frequency?

-





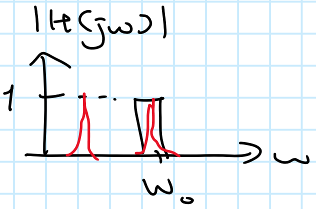
- Power at the output $= \frac{V_{out}^2}{R}$, Power at the input $= \frac{V_{in}^2}{R}$

- When $V_{in} = 1V$, $P_{in} = \frac{1}{R}$, and when $V_{out} = \frac{1}{\sqrt{2}}$, $P_{out} = \frac{1}{2R}$

$$dB = 10\log_{10} P_{out}/P_{in} \Rightarrow \boxed{\text{At } \omega_c,\ P_{out} = \frac{1}{2}P_{in}}$$

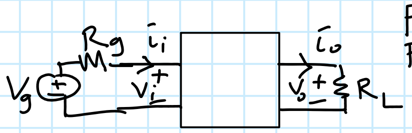
$$dB = 10\log_{10}(V_o/V_{in})^2$$

$$= 20\log_{10}(V_o/V_{in})$$



# P72

Thursday, May 18, 2023 9:39 AM



$P_i = V_i i_i$

$P_o = V_o i_o$

$\frac{P_o}{P_i} = \text{Gain}.$

Decibel (dB):

$$\boxed{\text{Gain in dB} = 10\log_{10}\frac{P_o}{P_i}}$$

If the gain, $G = 100$

$$\Rightarrow G_{dB} = 10\log_{10}100 = 10\log_{10}10^2 = 20\overset{1}{\cancel{\log_{10}10}}$$

$$= 20\ dB.$$

If $P_i$ is not given, $P_i = P_{ref} = 1W$.

$$G_{dB} = 10\log_{10}\frac{P_{out}}{1} = 10\log_{10}P_{out}$$

(dBm):

$$G_{dBm} = 10\log_{10}\frac{P_{out}}{1mW} = 10\log_{10}P_o \times 10^3$$

$$= 10\log_{10}P_o + 10\log_{10}10^3$$

$$\Rightarrow \boxed{G_{dBm} = G_{dB} + 30.}$$

Ex:

Find the decibel and milli-decibel of the following power gains?

a-) $G = 1000$

b-) $G = 10^{-6}$

Ans:

a-) $G_{dB} = 10\log_{10}10^3 = 30\ dB. \Rightarrow G_{dBm} = 60\ dBm.$

b-) $G_{dB} = -60\ dB \Rightarrow G_{dBm} = -30\ dBm.$

Ex:

A mobile phone has a sensitivity of

$S = -120\ dBm.$

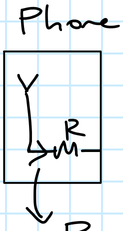
What does this imply?

# P73

Thursday, May 18, 2023 9:51 AM

Ans:

Sensitivity is the received power by the phone.



$P_{in} = -120\ dBm.$

The power in Watts is

$$P_{in_{dBm}} = P_{in_{dB}} + 30$$

$$P_{in_{dBm}} = -120 = P_{in_{dB}} + 30$$

$$\Rightarrow P_{in_{dB}} = -150.$$

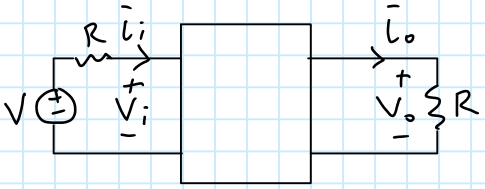
By definition,

$$P_{in_{dB}} = 10 \log_{10} \frac{P_{in}}{1}$$

$$-15\cancel{0} = \cancel{10} \log_{10} P_{in} \quad \Rightarrow P_{in} = 10^{-15}\ W = 0.001\ pW.$$

$$-15 = \log_{10} P_{in}$$

Also,



$$G = \frac{P_o}{P_i}, \quad G_{dB} = 10 \log_{10} P_o/P_i = 10 \log_{10} \frac{V_o^2/R}{V_i^2/R}$$

$$= 10 \log_{10} \left(\frac{V_o}{V_i}\right)^2$$

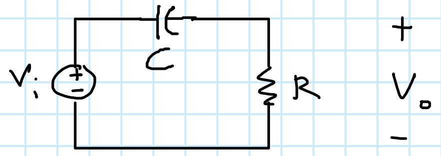
$$\boxed{G_{dB} = 20 \log_{10} \frac{V_o}{V_i}}$$

# P74

Thursday, May 18, 2023 10:11 AM

## 2-) High-Pass Filter (HPF)

- This circuit passes high frequencies, and blocks (filters out) low frequencies
- We can implement an HPF from a serial RC circuit as:



The transfer function is.

$$H(s) = \frac{R}{R + \frac{1}{sC}} = \frac{R}{\frac{1 + sRC}{sC}} = \frac{sRC}{1+sRC}$$

$$= \frac{sRC}{1+sRC} \cdot \frac{(1/RC)}{(1/RC)} = \frac{s}{s + \frac{1}{RC}}$$

Replace $s = j\omega \Rightarrow H(j\omega) = \frac{j\omega + 0}{j\omega + \frac{1}{RC}}$

$$\boxed{|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}}$$

$$H(j\omega) = \frac{\omega . e^{j(\tan^{-1}\frac{\omega}{0})}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2} . e^{j(\tan^{-1}\omega RC)}} \Rightarrow \theta(j\omega) = \tan^{-1}(\infty) - \tan^{-1}(\omega RC)$$

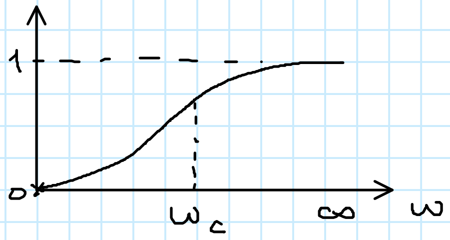
$$\boxed{\theta(j\omega) = 90^\circ - \tan^{-1}(\omega RC)}$$

# P75

Monday, May 31, 2021 11:23 AM

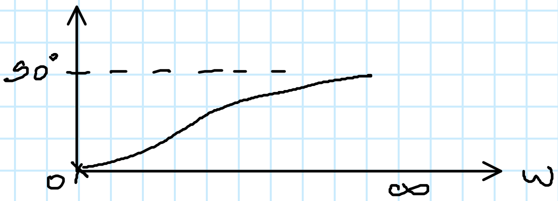
Bode Plots:

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

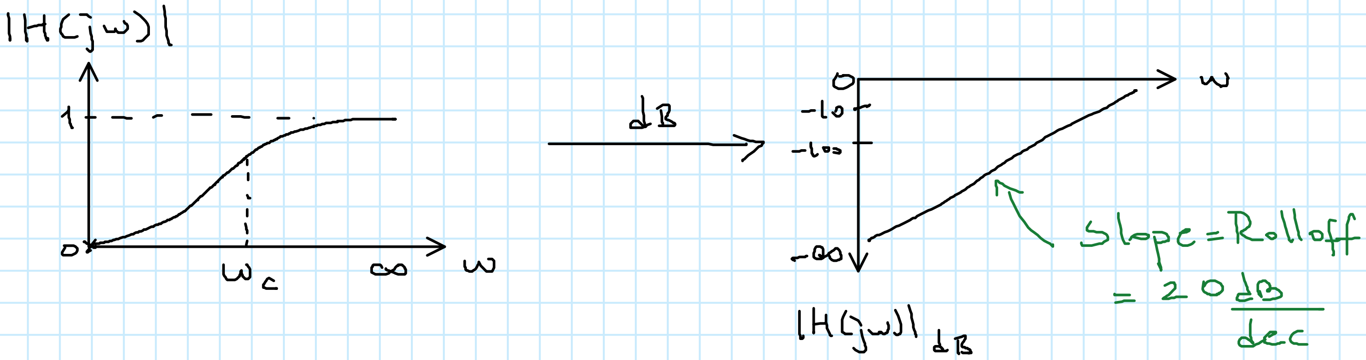


where $\omega_c = \frac{1}{RC}$ and $\boxed{f_c = \frac{1}{2\pi RC}}$

$$\theta(j\omega) = 90^\circ - \tan^{-1}\left(\frac{1}{\omega RC}\right)$$



- When drawing Bode plots, the axes are taken as decibels.

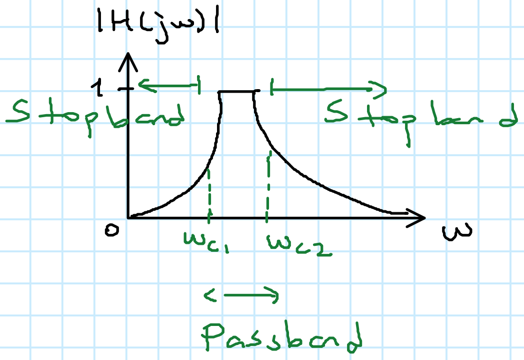


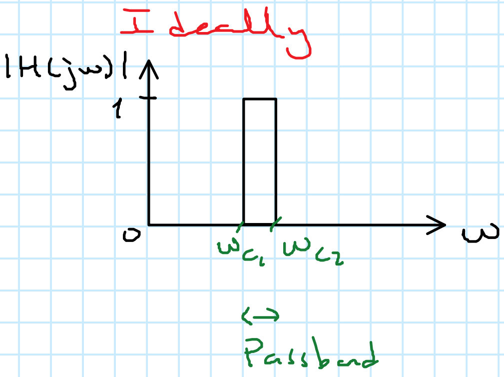
dec = decade = 10

- Roll-off is an indicator of filter selectivity
- With every degree of a filter roll-off increases by $20 \frac{dB}{dec}$

Roll-off = $\frac{40\,dB}{dec}$.

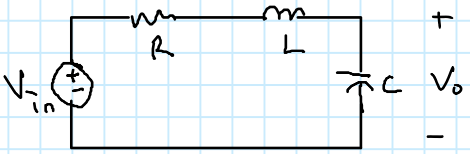
## 3-) Band-pass Filter (BPF)





- Implementation can be done by using LPF and HPF in series. or using an RLC circuit

For a serial RLC circuit:



$$H(s) = \frac{\left(\frac{R}{C}\right)s}{s^2 + \left(\frac{R}{L}\right)s + \left(\frac{1}{LC}\right)}$$

$$|H(j\omega)| = \frac{\omega(R/C)}{\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \left[\omega\left(\frac{R}{C}\right)\right]^2\right]^{1/2}}$$

$$\theta(j\omega) = 50^\circ - \tan^{-1}\left(\frac{\omega(R/L)}{\left(\frac{1}{LC}\right) - \omega^2}\right)$$

## HW#10

1-) In Proteus, generate 3 voltage signals at 3 frequencies and sum them.

$f_1$ = Sine wave at $f = 1$ kHz.

$f_2$ = " " at $f = 5$ kHz

$f_3$ = " " " $f = 15$ kHz.

Amplitudes are 1V.

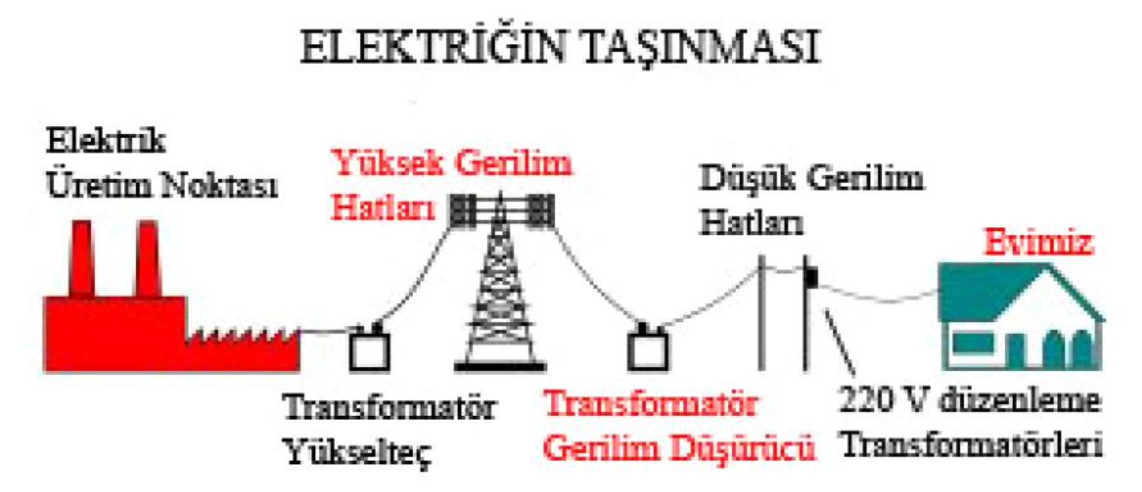
a-) Use a LPF to pass $f_1$ only. Show $|H(j\omega)|$.

b-) Use a HPF to " $f_3$ only. Show $|H(j\omega)|$.

c-) Use a BPF to pass $f_2$ only. " " .

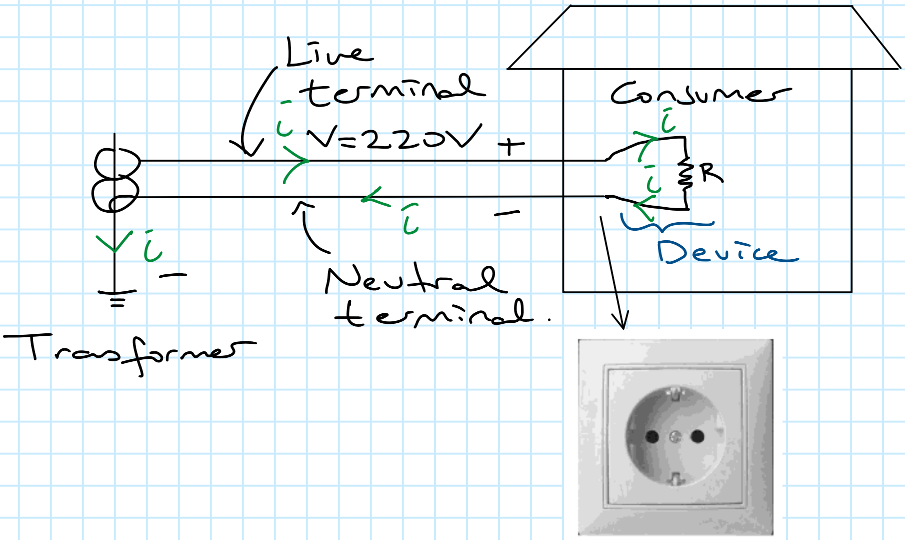
Use the filters with # of poles = 3, 5, 10.

# - Electrical Energy Distribution -



Consumer

## Consumer:

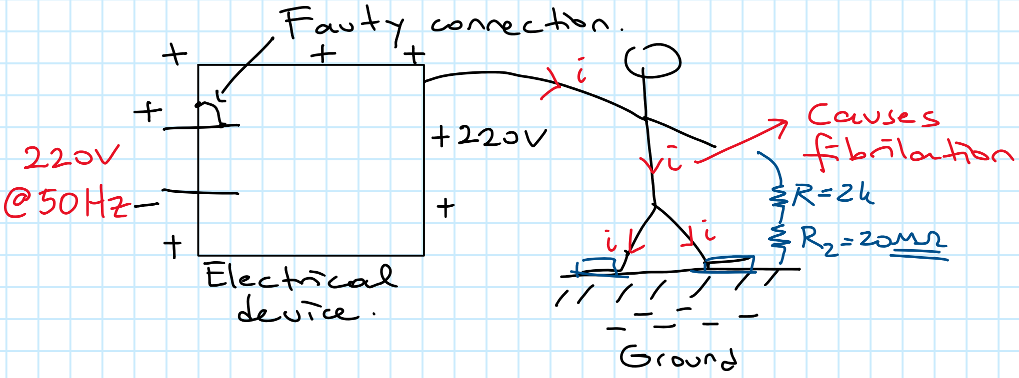


Live terminal = Phase terminal

## Electric Shock:

Current passing through human body.

It usually occurs as follows:



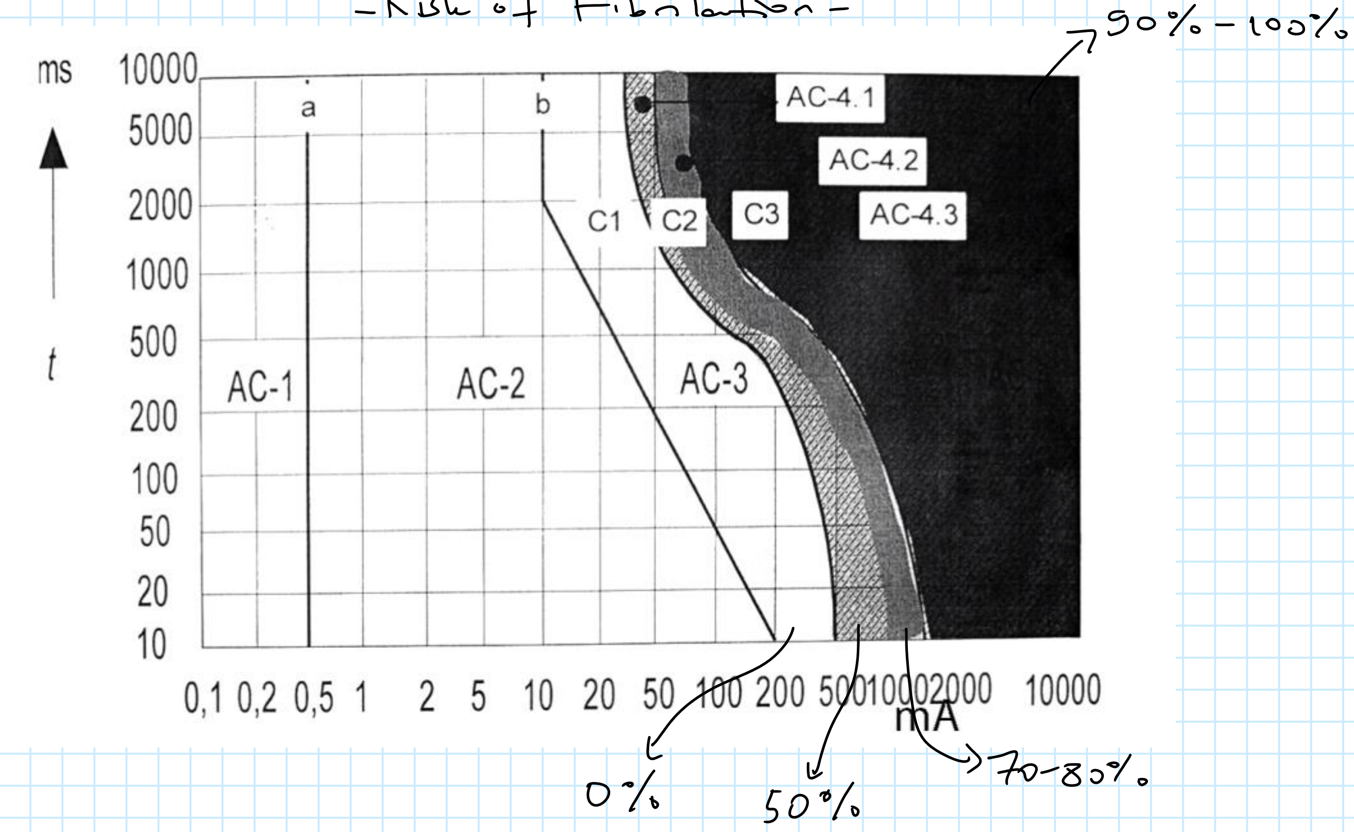
# P78

Thursday, May 25, 2023 10:51 AM

Fibrilation: The heart stops beating.

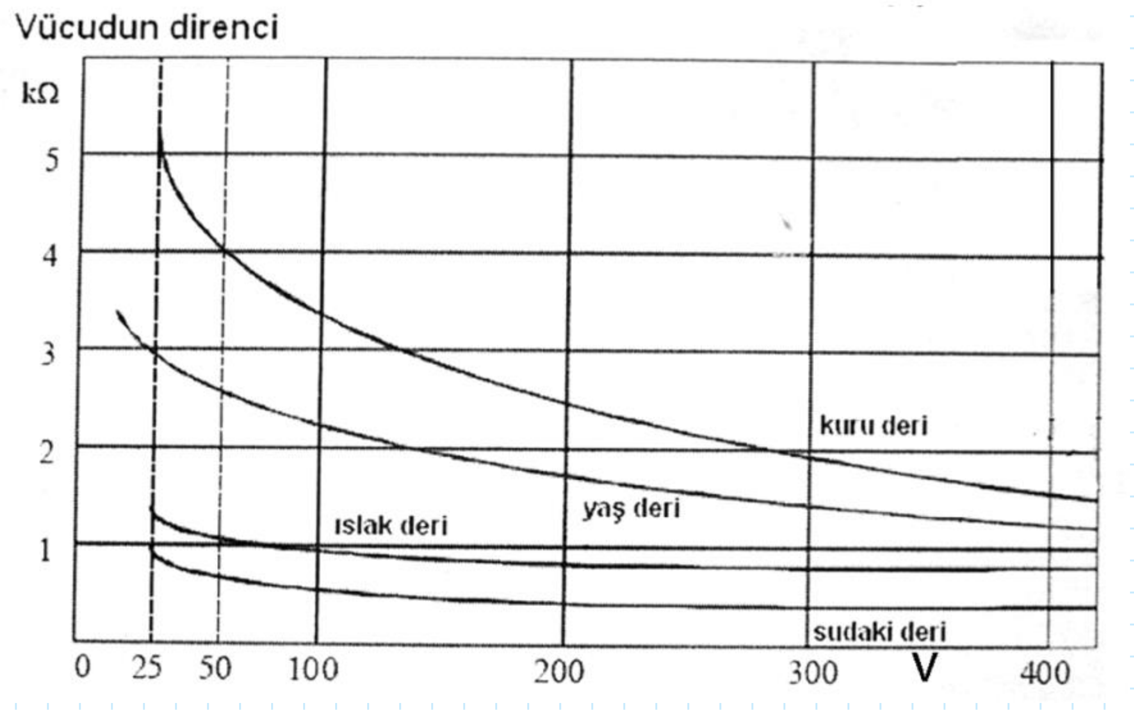
$I_{amp} \geq 30mA$ always poses the risk of fibrilation.

-Risk of Fibrilation-



$R_{human} \approx 2k\Omega$ , If $V = 220V$

$$\Rightarrow I = \frac{V}{R} = \frac{220V}{2200} = 0.1 = 100mA.$$



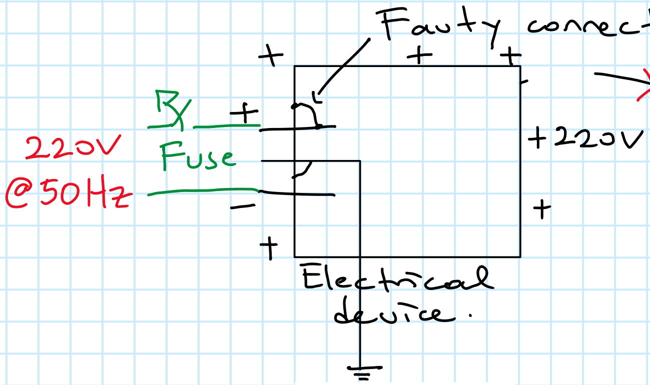
# P79

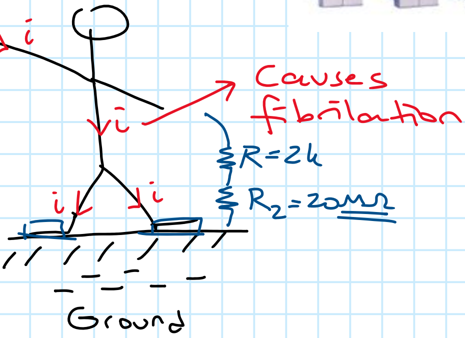
Thursday, May 25, 2023 11:08 AM

## Precautions:

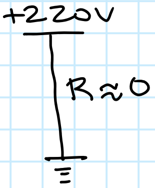
1-) Cutting-off the voltage source.
2-) Isolation
3-) Grounding:

Fuses: Switch



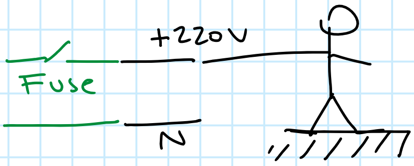


Fuses.



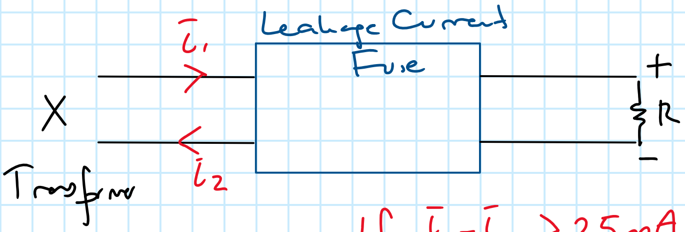
Short Circuit.

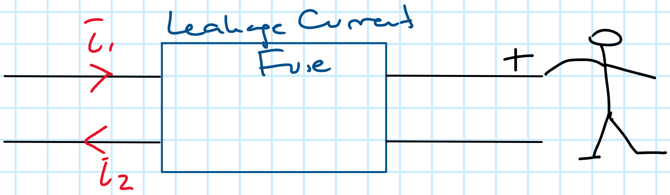
## Direct Contact Senario:



## Leakage Current Fuses:

The Major difference of Leakage current fuses from the standart fuses is that they protect from a direct contact electric shock as well.





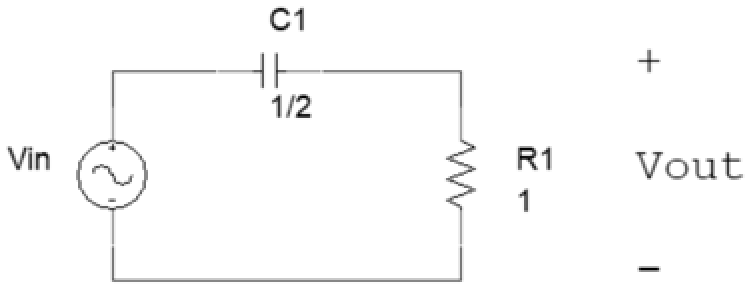
If $i_1 - i_2 \geqslant 25mA$ (or 30mA)
It blows.

# P80

Thursday, June 1, 2023 12:19 PM

Final Exam:
1-) Online.
2-) Calculator is allowed. Phone and computer use is not allowed.
3-) From the notes you may study the examples on P66 and

Given the following circuit (zero initial conditions)



Answer questions 1-5.

1-) What type of filter is this circuit ?

a-) LPF
b-) HPF
c-) BPF
d-) BRF

2-) Find its transfer function ?

[✓ a-)] $H(s) = \frac{s}{s+2}$
b-) $H(s) = \frac{1}{s+2}$
c-) $H(s) = \frac{2}{1+s}$
d-) $H(s) = \frac{1}{1+s/2}$

3-) Find the angle of the transfer function ?

a-) $\theta = -\tan^{-1}\omega/2$
b-) $\theta = -\tan^{-1}2\omega$
[✓ c-)] $\theta = \underbrace{\tan^{-1}\omega}_{\frac{\pi}{2}} - \tan^{-1}\omega/2$
d-) $\theta = -\tan^{-1}\omega - \tan^{-1}2\omega$

4-) Find the cut-off frequency $\omega_c$ (rad/sec) ?

a-) 0
b-) 1
[✓ c-)] 2
d-) 1/2

5-) Find $|H(j\omega)|$ at $\omega = 2\ rad/sec$ ?

a-) $\sqrt{2}$
b-) $2\sqrt{2}$
c-) 1/2
[✓ d)] $1/\sqrt{2}$

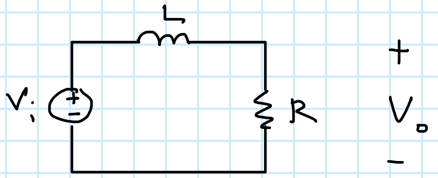
Note: This example is in test format. Your question will be in Text format.

# P81

Sunday, June 4, 2023 8:05 PM

- Final Exam Preparation -

1-)



✓Find $H(s)=?$

Find the type of the filter

✓Find $|H(j\omega)|=?$

✓ Find $\angle H(j\omega)=?$

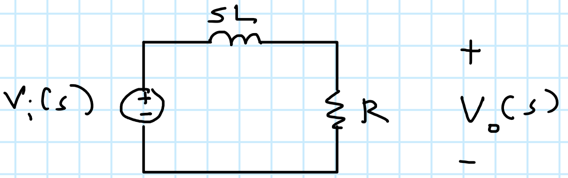
Find the corner freq?

Ans:

**Step 1**: Find the transfer function.

$$H(s)=\frac{V_o(s)}{V_i(s)}$$

In s-domain



$$\Rightarrow H(s)=\frac{V_o(s)}{V_i(s)}$$

$$\Rightarrow V_o(s)=V_i(s)\cdot\frac{R}{R+sL}$$

$$\Rightarrow H(s)=\frac{R}{sL+R}\,\frac{(\frac{1}{L})}{(\frac{1}{L})}=\frac{R/L}{s+R/L}$$ ← Proper form.

**Step 2**: Replace $s=j\omega$ and obtain the $H(j\omega)$

$$H(j\omega)=\frac{R/L}{R/L+j\omega}$$

**Step 3**: Obtain polar form for $H(j\omega)$

$$H(j\omega)=\frac{R/L\,e^{j0}}{\sqrt{\omega^2+\left(\frac{R}{L}\right)^2}\;e^{j\tan^{-1}\frac{\omega}{R/L}}}$$

**Step 4**: Obtain

$|H(j\omega)|$ $\angle H(j\omega)$

$$|H(j\omega)|=\frac{R/L}{\sqrt{\omega^2+\left(\frac{R}{L}\right)^2}}\quad,\quad \angle H(j\omega)=\underbrace{\left(0-\tan^{-1}\frac{\omega L}{R}\right)}_{\theta}$$

# P82

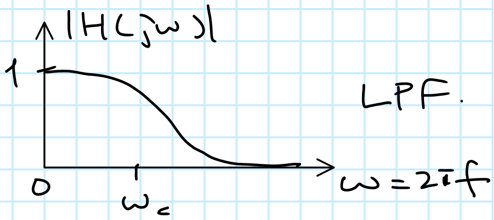
Sunday, June 4, 2023 8:20 PM

**Type of the filter:**

$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$$

Put $\omega = 2\pi f = 0$ , $|H(j\omega)| = 1$

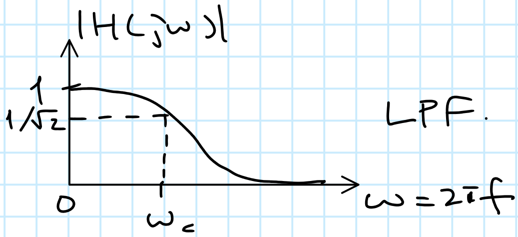
Put $\omega = \infty$ , $|H(j\omega)| = 0$



LPF.

Corner Frequency:

$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}} = \frac{1}{\sqrt{2}}$$



LPF.

$$\frac{R^2/L^2}{\omega_c^2 + \frac{R^2}{L^2}} = \frac{1}{2}$$

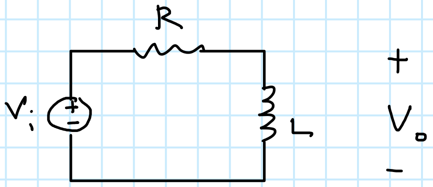
$$\cancel{2}\frac{R^2}{L^2} = \omega_c^2 + \cancel{\frac{R^2}{L^2}}$$

$$\Rightarrow \omega_c = \frac{R}{L} \Rightarrow f_c = \frac{1}{2\pi}\frac{R}{L} \text{ (Hz)}$$

# P83

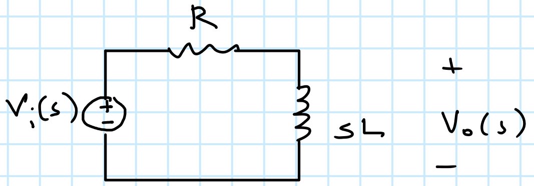
Sunday, June 4, 2023 8:26 PM

2-)



Solve this filter.

Ans:



$$\Rightarrow H(s) = \frac{sL}{sL+R}$$

Proper form: $H(s) = \frac{sL}{sL+R}\,\frac{(\frac{1}{L})}{(\frac{1}{L})} = \frac{s}{s+R/L}$

$$H(j\omega) = \frac{0+j\omega}{j\omega + R/L}$$

or

$$H(j\omega) = \frac{\omega e^{j\tan^{-1}\infty}}{\sqrt{\omega^2+(\frac{R}{L})^2}\, e^{j\tan^{-1}\frac{\omega}{R/L}}}$$

$|H(j\omega)|$

Then,

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2+(\frac{R}{L})^2}}, \quad \angle H(j\omega) = \tan^{-1}\infty - \tan^{-1}\frac{\omega L}{R}$$

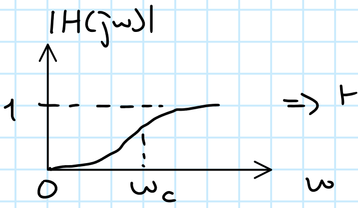
$$= \frac{\pi}{2} - \tan^{-1}\frac{\omega L}{R}$$

Type of the filter:

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2+(\frac{R}{L})^2}}$$

Put $\omega = 0$, $|H(j\omega)| = 0$

Put $\omega = \infty$, $|H(j\omega)| = 1$



⇒ HPF.

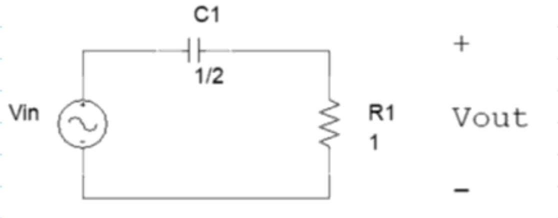
Corner Frequency:

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2+(\frac{R}{L})^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{\omega_c^2}{\omega_c^2+(\frac{R}{L})^2} = \frac{1}{2} \Rightarrow \cancel{2}\omega_c^2 = \cancel{\omega_c^2} + \left(\frac{R}{L}\right)^2$$

$$\Rightarrow \omega_c = \frac{R}{L}, \; f_c = \frac{1}{2\pi}\,\frac{R}{L}$$

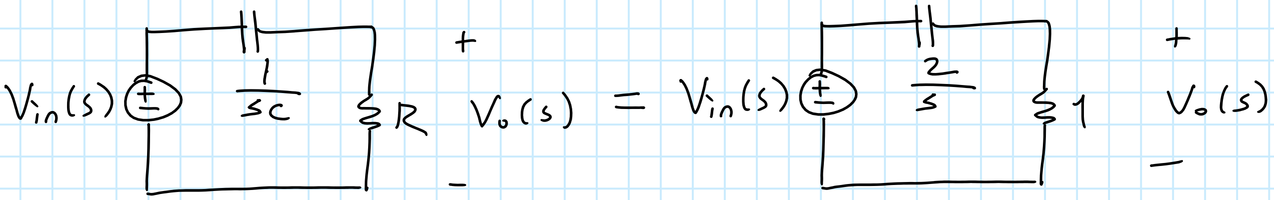
3-)



Analyze this filter.

Ans:

In s-domain



$$\Rightarrow \quad H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1+\frac{2}{s}} = \frac{1}{\frac{s+2}{s}} = \frac{s}{s+2}$$

Then,

$$H(j\omega) = \frac{j\omega}{j\omega + 2}$$

In polar form,

$$H(j\omega) = \frac{\omega e^{j\frac{\pi}{2}}}{\sqrt{\omega^2 + 2^2}\, e^{j\tan^{-1}\frac{\omega}{2}}}$$

Then,

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2+4}} \quad , \quad \angle H(j\omega) = \frac{\pi}{2} - \tan^{-1}\frac{\omega}{2}$$

The cut-off freq:

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2+4}} = \frac{1}{\sqrt{2}}$$

$$\frac{\omega_c^2}{\omega_c^2+4} = \frac{1}{2} \Rightarrow \cancel{2}\omega_c^2 = \cancel{\omega_c^2} + 4$$

$$\omega_c^2 = 4$$

$$\omega_c = 2 \text{ rad/s.}$$

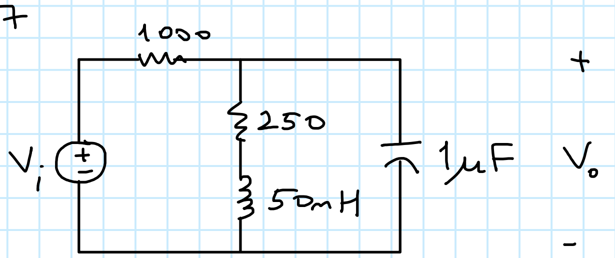
# P85

Sunday, June 4, 2023 8:55 PM

4)

Ex:

Given the circuit

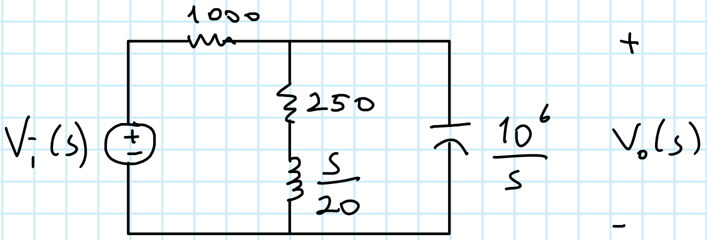


$V_g = 50t\,u(t)$

Find $H(s)$.

Find $V_o(t)$ using $H(s)$.

Ans:



$$\left(\frac{10^6}{s}\right) \parallel \underbrace{\left(250+\frac{s}{20}\right)}_{\frac{5000+s}{20}} = \frac{\frac{10^6}{s} \times \frac{5000+s}{20}}{\underset{(20)}{\frac{10^6}{s}} + \underset{(s)}{\frac{5000+s}{20}}} = \frac{\frac{5\times10^9+10^6 s}{\cancel{20s}}}{\frac{20\times10^6+s^2+5000s}{\cancel{20s}}}$$

$$= \frac{10^6 s + 5\times10^9}{s^2+5000s+2\times10^7}$$

$$H(s) = \frac{\frac{10^6 s + 5\times10^9}{s^2+5000s+2\times10^7}}{\frac{10^6 s + 5\times10^9}{s^2+5000s+2\times10^7} + 1000} = \frac{\frac{10^6 s + 5\times10^9}{\cancel{s^2+5000s+2\times10^7}}}{\frac{10^6 s + 5\times10^9 + 1000s^2 + 5\times10^6 s + 2\times10^{10} \xrightarrow{} 20\times10^9}{\cancel{s^2+5000s+2\times10^7}}}$$

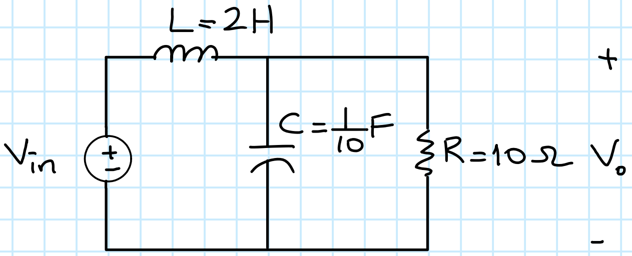
$$\frac{10^6 s + 5\times10^9}{1000s^2+6\times10^6 s+25\times10^9} \quad \frac{\left(\frac{1}{1000}\right)}{\left(\frac{1}{1000}\right)}$$

$$= \frac{10^3 s + 5\times10^6}{s^2+6000s+25\times10^6} = \frac{1000(s+5000)}{s^2+6000s+25\times10^6}$$

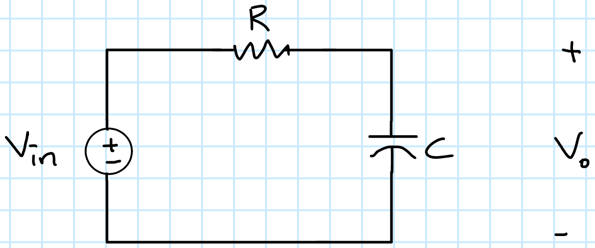
# P86

Monday, June 5, 2023 9:54 AM

1. Find the transfer function H(s) for the following circuit.

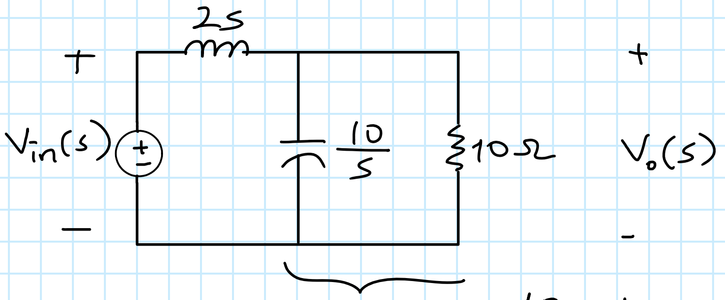


2. For the filter circuit shown below, find
   a. the expression for the transfer function, H(jω).
   b. the type of this filter.
   c. the expression for the cut-off frequency (corner frequency).
   d. the phase expression of the transfer function.



Answers :

1-) In s-domain,

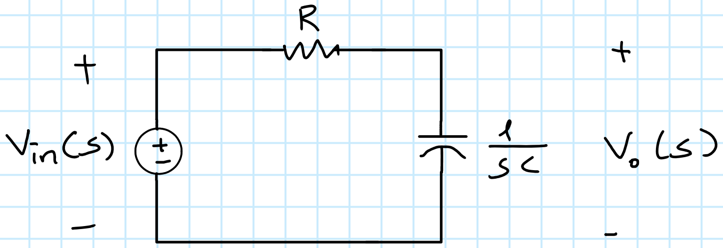


$$\frac{10}{s} \| 10 = \frac{\frac{10}{s}\times 10}{\frac{10}{s}+10} = \frac{\frac{100}{\cancel{s}}}{\frac{10+10s}{\cancel{s}}}$$

$$= \frac{1\cancel{0}0}{1\cancel{0}(s+1)} = \frac{10}{s+1}$$

$$\Rightarrow H(s) = \frac{\frac{10}{s+1}}{2s+\frac{10}{s+1}} = \frac{\frac{10}{\cancel{s+1}}}{\frac{2s^2+2s+10}{\cancel{s+1}}} = \frac{10}{2s^2+2s+10} = \frac{5}{s^2+s+5}$$

2-) In s-domain,



$$\Rightarrow H(s) = \frac{\frac{1}{sC}}{\frac{1}{sC}+R} = \frac{\frac{1}{\cancel{sC}}}{\frac{1+sRC}{\cancel{sC}}} = \frac{1}{1+sRC} = \frac{1/RC}{s+1/RC}$$

Then

$$H(j\omega) = \frac{1/RC}{j\omega + 1/RC}$$

In polar form,

$$H(j\omega) = \frac{1/RC}{\sqrt{\omega^2+\left(\frac{1}{RC}\right)^2}\; e^{j\tan^{-1}\frac{\omega}{1/RC}}}$$

# P88

Monday, June 5, 2023 10:25 AM

Then,

$$|H(j\omega)| = \frac{1/RC}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\omega RC.$$

The corner frequency:

$$\frac{1/RC}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{\left(\frac{1}{RC}\right)^2}{\omega_c^2 + \left(\frac{1}{RC}\right)^2} = \frac{1}{2} \Rightarrow \cancel{2}\left(\frac{1}{RC}\right)^2 = \omega_c^2 + \cancel{\left(\frac{1}{RC}\right)^2}$$

$$\omega_c^2 = \left(\frac{1}{RC}\right)^2$$

$$\omega_c = \frac{1}{RC} \Rightarrow f_c = \frac{1}{2\pi RC}\text{ Hz}.$$