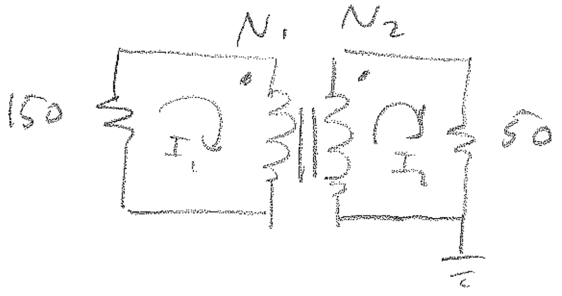
**Ex:**

Match 150 Ω dipole antenna to a 50 Ω coaxial cable using a transformer balun.

Find $(N_1 : N_2)$ — Turn ratio of the primary to secondary windings.

**Ans:**




$N_1 : N_2 = ?$

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \quad \text{and} \quad N_1 I_1 = N_2 I_2$$

$$Z_{in} = \frac{1}{a^2} Z_L$$

$$\Rightarrow 150 = \frac{1}{a^2} 50 \Rightarrow \boxed{a^2 = \frac{1}{3}}$$

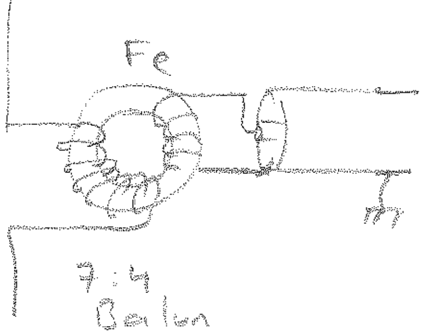
$$\frac{V_1}{V_2} = \frac{1}{a} = \frac{N_1}{N_2} \Rightarrow \frac{N_1}{N_2} = \sqrt{3}$$

$$\Rightarrow N_1 : N_2 = \sqrt{3} : 1$$

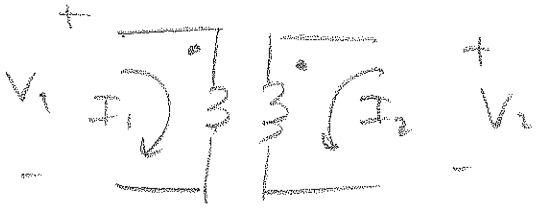
$$\Rightarrow N_1 : N_2 \simeq 1.75 : 1$$

or

$$\boxed{N_1 : N_2 = 7 : 4}$$

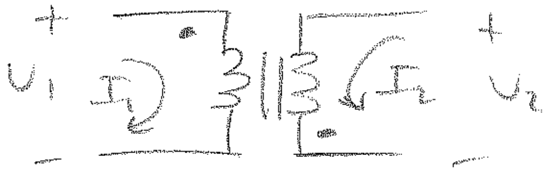


## Note:



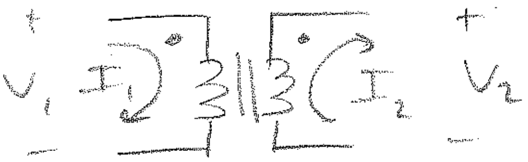

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$$N_1 I_1 = -N_2 I_2$$



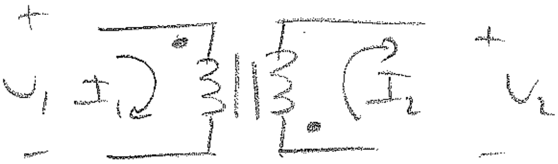

$$\frac{V_1}{N_1} = \frac{-V_2}{N_2}$$

$$N_1 I_1 = N_2 I_2$$




$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$$N_1 I_1 = N_2 I_2$$



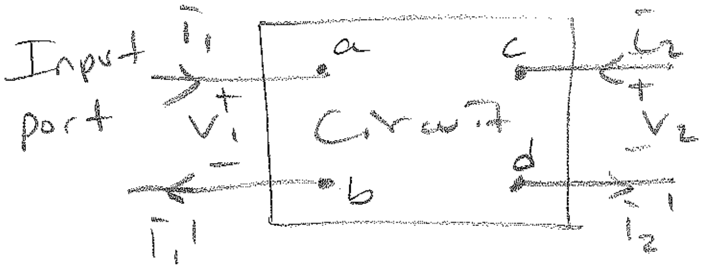

$$\frac{V_1}{N_1} = \frac{-V_2}{N_2}$$

$$N_1 I_1 = -N_2 I_2$$

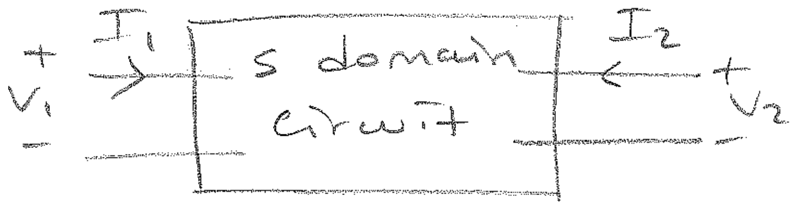
—o—

# – Two-Port Circuits –

Two port circuits are defined by the currents and voltages at two ports of the circuit.




or




## 2 port equations:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}_{Z\text{-parameters}} \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{I}$$

or

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{12} V_1 + Y_{22} V_2$$

Y-parameters.

or

$$V_2 = b_{11} V_1 - b_{12} I_1$$
$$I_2 = b_{21} V_1 - b_{22} I_1$$

b-param.

$$\left.\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right\} \text{h-param.}$$

or

$$\left.\begin{aligned} I_1 &= g_{11} V_1 + g_{12} I_2 \\ V_2 &= g_{21} V_1 + g_{22} I_2 \end{aligned}\right\} \text{g-param.}$$

---

<u>z - Parameters:</u>

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega \quad \text{(Input Impedance)}$$

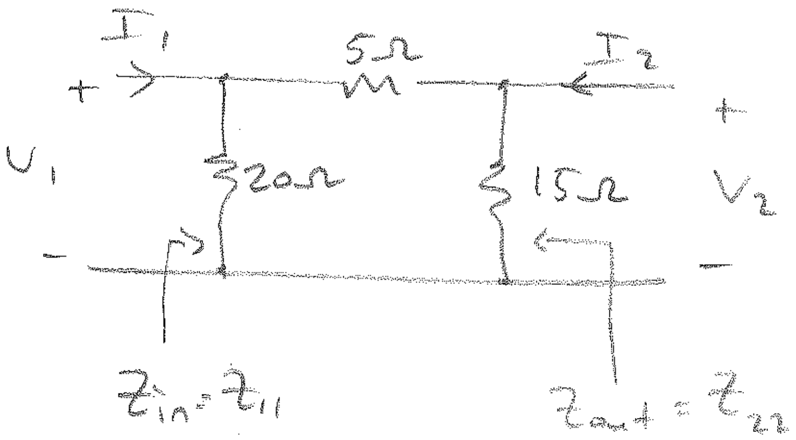
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega \quad \text{(Forward Transfer Impedance)}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega \quad \text{(Reverse Transfer Impedance)}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega \quad \text{(Output Impedance)}$$

Ex:

Given the circuit




Find the z-parameters of this 2-port circuit?

Ans:

Purely resistive circuit, thus it is also s-domain circuit.

$$\Rightarrow Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{20 \cdot 20}{40} = 10\,\Omega$$

($I_2 = 0$ → open circuit)

when $I_2 = 0$

$$V_2 = \frac{V_1}{15+5}(15) = 0.75\,V_1$$

$$\Rightarrow Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{0.75\,V_1}{\frac{V_1}{10}} = 7.5\,\Omega$$

$$\Rightarrow Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{(15)(25)}{40} = 9.375\,\Omega$$

When port 1 is open,

$$V_1 = \frac{V_2}{5+20}(20) = 0.8V_2 \quad \text{and} \quad I_2 = \frac{V_2}{9.375}$$

Thus,

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{0.8\,V_2}{\frac{V_2}{9.375}} = 7.5\,\Omega$$

$$\Rightarrow Z = \begin{bmatrix} 10 & 7.5 \\ 7.5 & 9.375 \end{bmatrix}\Omega.$$

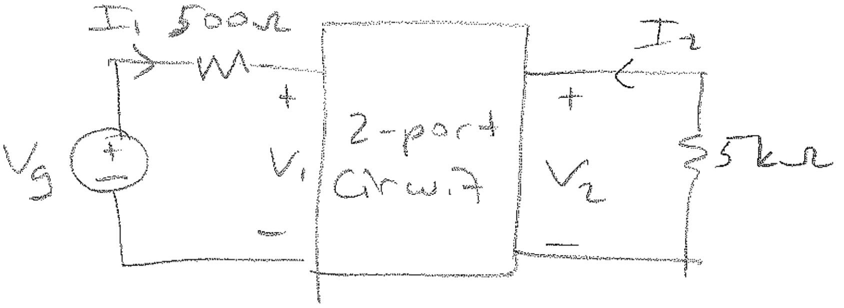
<u>Analysis of the Terminated Two-Port Circuit:</u>

The important parameters of the terminated two-port circuits are:

- Input impedance $Z_{in} = \frac{V_1}{I_1}$ or the admittance $Y_{in} = \frac{I_1}{V_1}$
- The output current $I_2$.
- Thevenin voltage and impedance ($V_{th}$, $R_{th}$) with respect to port 2.
- The current gain $\frac{I_2}{I_1}$.
- The voltage gain $\frac{V_2}{V_1}$.
- The voltage gain $\frac{V_2}{V_g}$.

Ex:

The two-port circuit is shown below.




$V_g = 500\angle 0^\circ$

Given $b_{11} = -20$

$b_{12} = -3000\Omega$

$b_{21} = -2mS$

$b_{22} = -0.2$

Q1-)

Find the phasor voltage $V_2$?

Ans:

$V_2 = I_2 (5k\Omega)$ $\Rightarrow I_2 = \dfrac{-V_g \Delta b}{b_{11}Z_g + b_{21}Z_g Z_L + b_{22}Z_L + b_{12}}$ (From the table given)

$I_2 = ?$

where $Z_g = 500\Omega$, $Z_L = 5k\Omega$

and $\Delta b = (-20)(-0.2) - (-3000)(-2\times10^{-3})$
$= -2$

Substitute all the parameters into the equation

$I_2 = \frac{1}{19} A.$

$\Rightarrow V_2 = -I_2(5000) = 263.16\angle 0^\circ$

Q2-1

Find the average power delivered to the 5kΩ load?

Ans:

$$P = \frac{V_2^2}{2R} = \frac{263.16^2}{2(5000)} = 6.93\,W$$

Q3:

Find the average power delivered to the input port?

Ans:

$$P_1 = \frac{1}{2} I_1^2 \cdot Z_{in}$$ (From the table.)

where

$$Z_{in} = \frac{b_{22} Z_L + b_{12}}{b_{21} Z_L + b_{11}} = 133.33\,\Omega$$

$$I_1 = \frac{500}{500 + 133.33} = 785.47\,mA$$

$$\Rightarrow P_1 = \frac{1}{2}(785.47 \times 10^{-3})^2 \cdot (133.33) = \underline{44.55\,W}.$$

$, z_{12} = z_{21} = 17\ \Omega,$

## l Two-Port Circuit

he circuit is driven at port 1
domain circuit diagram for
$Z_g$ represents the internal
e of the source, and $Z_L$ the
es expressing the terminal
-port parameters, $V_g$, $Z_g$,

rt circuit define its terminal

lmittance $Y_{in} = I_1/V_1$

$Z_{Th}$) with respect to port 2

### arameters

erived, we develop the ex-
e two-port portion of the
involving the $y$, $a$, $b$, $h$,



**TABLE 18.2 Terminated Two-Port Equations**

| z PARAMETERS | y PARAMETERS |
|---|---|
| $Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$ | $Y_{in} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$ |
| $I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$ | $I_2 = \frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$ |
| $V_{Th} = \frac{z_{21}}{z_{11} + Z_g}V_g$ | $V_{Th} = \frac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$ |
| $Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$ | $Z_{Th} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$ |
| $\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$ | $\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$ |
| $\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$ | $\frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$ |
| $\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$ | $\frac{V_2}{V_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_g Z_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$ |

| a PARAMETERS | b PARAMETERS |
|---|---|
| $Z_{in} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$ | $Z_{in} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$ |
| $I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_g Z_L + a_{22}Z_g}$ | $I_2 = \frac{-V_g \Delta b}{b_{11}Z_g + b_{21}Z_g Z_L + b_{22}Z_L + b_{12}}$ |
| $V_{Th} = \frac{V_g}{a_{11} + a_{21}Z_g}$ | $V_{Th} = \frac{V_g \Delta b}{b_{22} + b_{21}Z_g}$ |
| $Z_{Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$ | $Z_{Th} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$ |
| $\frac{I_2}{I_1} = \frac{-1}{a_{21}Z_L + a_{22}}$ | $\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L}$ |
| $\frac{V_2}{V_1} = \frac{Z_L}{a_{11}Z_L + a_{12}}$ | $\frac{V_2}{V_1} = \frac{\Delta b Z_L}{b_{12} + b_{22}Z_L}$ |
| $\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$ | $\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$ |

| h PARAMETERS | g PARAMETERS |
|---|---|
| $Z_{in} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$ | $Y_{in} = g_{11} - \frac{g_{12}g_{21}}{g_{22} + Z_L}$ |
| $I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L}$ | $I_2 = \frac{-g_{21}V_g}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$ |
| $V_{Th} = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h}$ | $V_{Th} = \frac{g_{21}V_g}{1 + g_{11}Z_g}$ |
| $Z_{Th} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h}$ | $Z_{Th} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$ |
| $\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$ | $\frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$ |
| $\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{\Delta h Z_L + h_{11}}$ | $\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L}$ |
| $\frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$ | $\frac{V_2}{V_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$ |