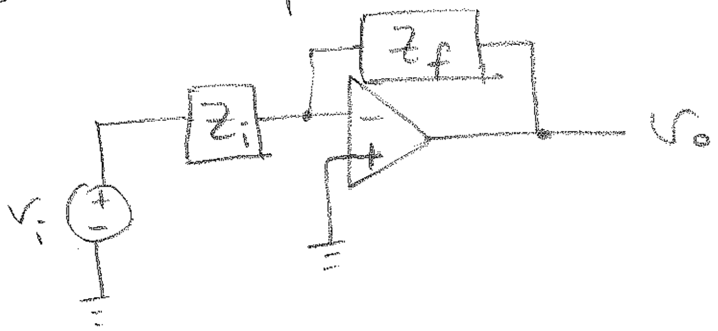
Active Filters:

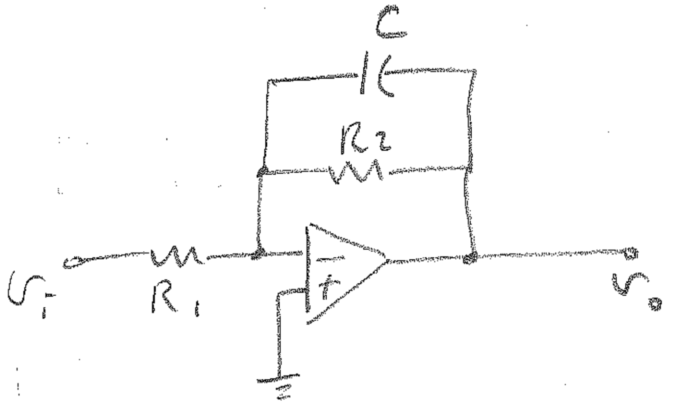
General op-amp filter circuit is:



$$\Rightarrow H(s) = \frac{-Z_f}{Z_i}$$

Low-Pass Filter:

Consider



$$H(s) = \frac{-R_2 \,\|\, \left(\frac{1}{sC}\right)}{R_1}$$

or $\boxed{H(s) = -k\,\frac{\omega_c}{s+\omega_c}}$ where $\boxed{k = \frac{R_2}{R_1}}$, $\boxed{\omega_c = \frac{1}{R_2 C}}$

Ex:

Find C and R_2 given that $R_1 = 1\Omega$ to have a LPF with a gain of 1 and cutoff frequency of 1 rad/sec.

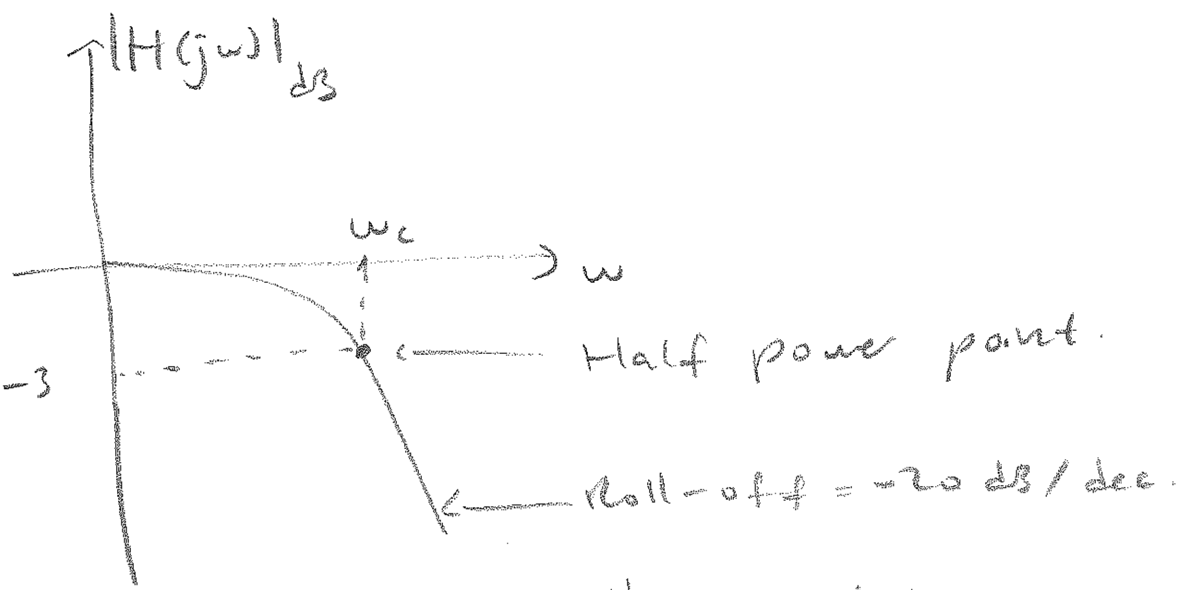
Ans:

$k = \frac{R_2}{R_1} \rightarrow R_2 = kR_1$, $\omega_c = \frac{1}{R_2 C} \rightarrow C = \frac{1}{R_2 \omega_c}$

$R_2 = R_1$ or $\boxed{C = \frac{1}{1 \cdot 1} = 1F}$

$\boxed{R_2 = 1\Omega}$

$\Rightarrow H(s) = \frac{-1}{s+1}$, the minus sign affects the phase.

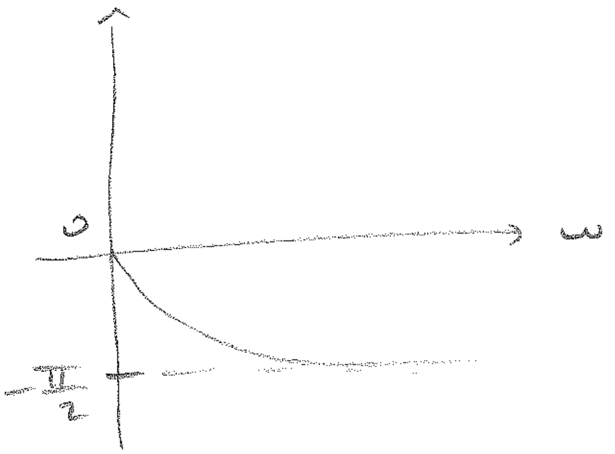


1^{st} order filter.

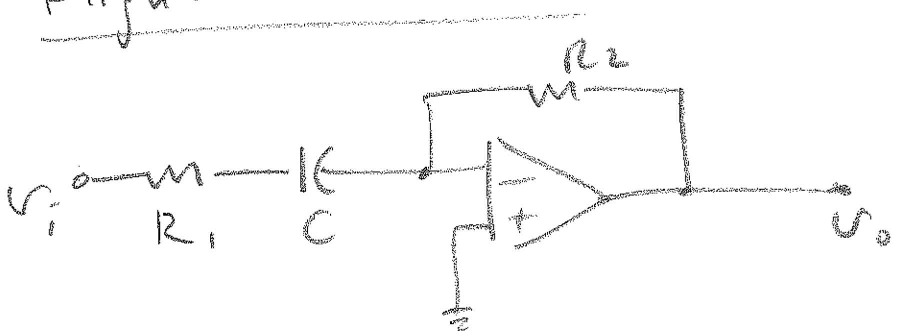
Also

$$H(j\omega) = \frac{1}{1+j\omega} = \underbrace{\frac{1}{\sqrt{\omega^2+1}}}_{|H(j\omega)|} \underbrace{\frac{e^{j\tan^{-1}0}}{e^{j\tan^{-1}\omega}}}_{\angle H(j\omega) = \theta(j\omega)}$$

$$\theta(j\omega) = \frac{\tan^{-1}0}{\tan^{-1}\omega} = \boxed{-\tan^{-1}\omega}$$



High-Pass Filter:



$$H(s) = \frac{-R_2}{R_1 + \frac{1}{sC}} = \boxed{-K \cdot \frac{s}{s+\omega_c}}$$

where $\boxed{K = \frac{R_2}{R_1} = \text{Gain}}$, $\boxed{\omega_c = \frac{1}{R_1 C}}$ = cutoff freq.

Band-Pass Filter:

This filter can be constructed as:

$$v_i \longrightarrow \underset{H_1(s)}{\boxed{\text{LPF}}} \longrightarrow \underset{H_2(s)}{\boxed{\text{HPF}}} \longrightarrow \underset{H_3(s)}{\boxed{\text{Inverting Amplifier}}} \longrightarrow v_o$$

Total transfer function:

$$H(s) = H_1(s)\,H_2(s)\,H_3(s)$$

$$= \left(\frac{-\omega_{c2}}{s+\omega_{c2}}\right)\left(\frac{-s}{s+\omega_{c1}}\right)\cdot\left(-\frac{R_f}{R_i}\right)$$

$$= -k\,\frac{\omega_{c2}\cdot s}{s^2+\omega_{c2}s+\omega_{c1}\omega_{c2}}$$

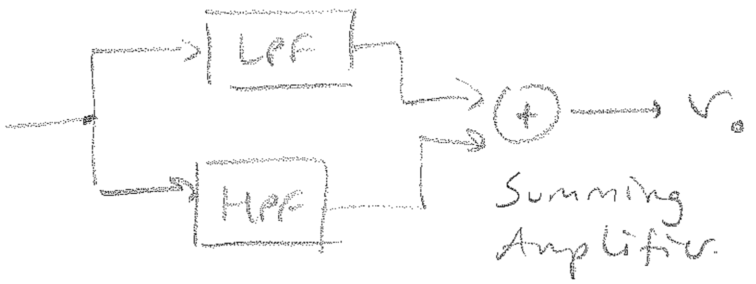
where

$$\omega_{c1} = \frac{1}{R_H C_H}$$

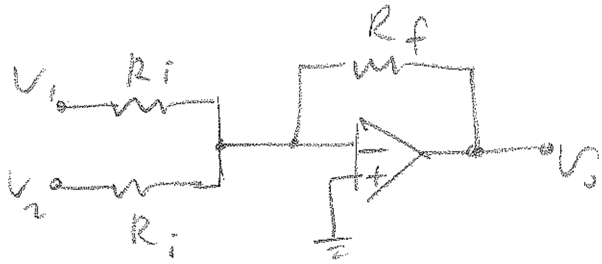
$$\omega_{c2} = \frac{1}{R_L C_L}$$

$$|H(j\omega)| = k = \frac{R_f}{R_i}$$

Band Reject Filter:



Summing Amplifier



$$H(s) = \frac{R_f}{R_i}\left[\frac{s^2 + 2\omega_{c_1} s + \omega_{c_1}\omega_{c_2}}{(s+\omega_{c_1})\cdot(s+\omega_{c_2})}\right]$$

where

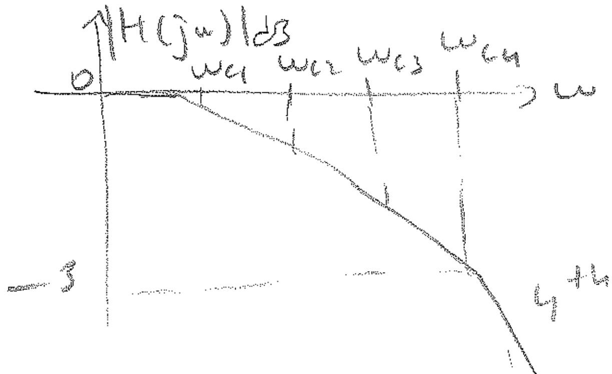
$$\omega_{c_1} = \frac{1}{R_L C_L}, \quad \omega_{c_2} = \frac{1}{R_H C_H}, \quad k = \frac{R_f}{R_i}$$

High-Order Filters:

$V_i \longrightarrow$ LPF $\longrightarrow$ LPF $\longrightarrow \cdots \longrightarrow$ LPF $\longrightarrow v_o$

$$H(s) = \left(\frac{-1}{s+1}\right)\left(\frac{-1}{s+1}\right)\cdots\left(\frac{-1}{s+1}\right) \quad (\text{for } \omega_c = 1 \text{ rad/sec})$$

$$\Rightarrow H(s) = \frac{(-1)^n}{(s+1)^n}$$ where n = # of filters used & order of the filter

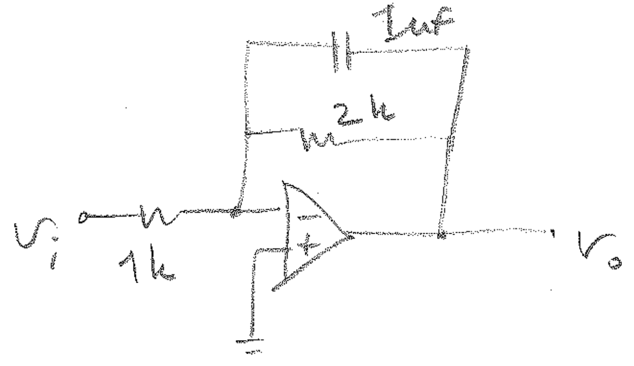


$\omega_{cn} = \sqrt{\sqrt[n]{2} - 1}$ (n^{th} order filter cut-off freq.)

4^{th} order (-80dB/dec roll-off)

Ex:

Given the circuit



① Find the corner freq?

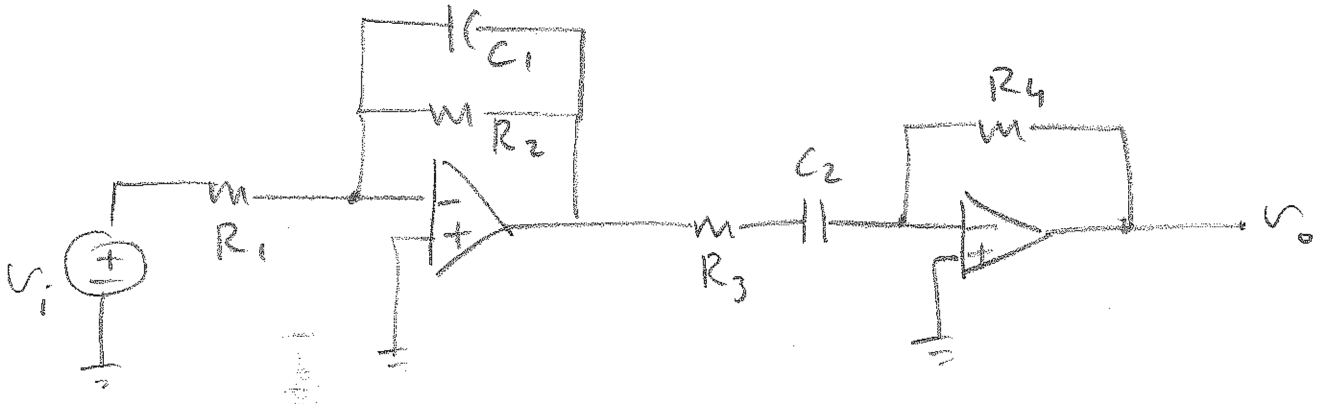
Ans: $\omega_c = \frac{1}{RC} = \frac{1}{(2k)(1\mu F)} = 500 \text{ rad/sec.}$

② Find the maximum gain?

Ans: $k = \frac{R_2}{R_1} = 2$

Ex:

Given the circuit



where $R_1 = R_2 = 80\,\Omega$, $C_1 = 0.2\,\mu F$

$R_3 = R_4 = 8\,k\Omega$, $C_2 = 0.2\,\mu f$.

Question 1:

What is the type of this filter?

a-) LP b-) BP c-) HP d-) BR (Band Reject)

Question 2:

Find the corner frequencies?

Ans:

LP Stage:

$$\omega_{c_2} = \frac{1}{R_2 C} = \frac{1}{(80)(0.2\times10^{-6})} = 20000\pi$$

$$2\pi f_{c_2} = 20000\pi$$

$$\boxed{f_{c_2} = 10\,kHz}$$

HP Stage:

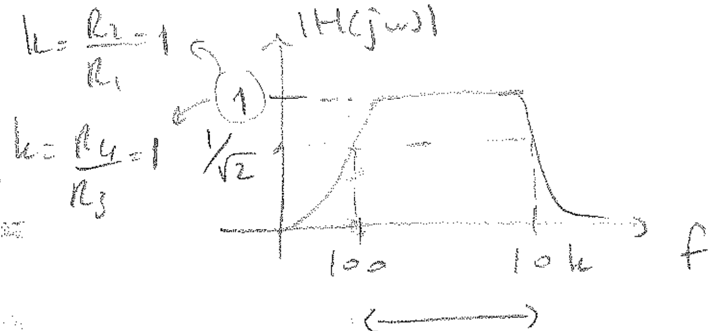
$$\omega_{c_1} = \frac{1}{R_4 C} = \frac{1}{(8000)(0.2\times10^{-6})} = 200\pi$$

$$\Rightarrow \boxed{f_{c_1} = 100\,Hz}$$

Question 3 :

Find the bandwidth of this filter?

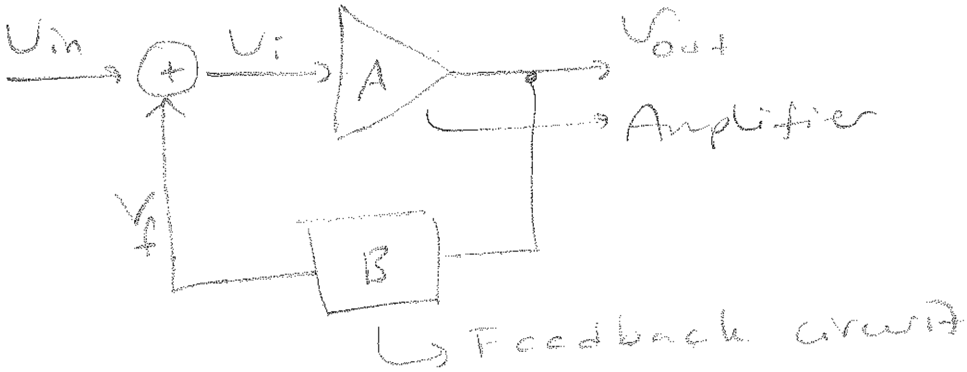
$$B = f_{C_2} - f_{C_1} = 10000 - 100 = 9900\ \text{Hz} = 9.9\ \text{kHz}.$$



B = Bandwidth = 9.9 kHz

— Oscillator Theory —

Consider the following circuit



We aim to find $\frac{V_o}{V_{in}}$ = Closed Loop Gain

We have

$V_o = A V_i$ ——— (1)

$V_f = B V_o$ ——— (2)

$V_{in} + V_f = V_i$ ——— (3)

Take eqn (3):

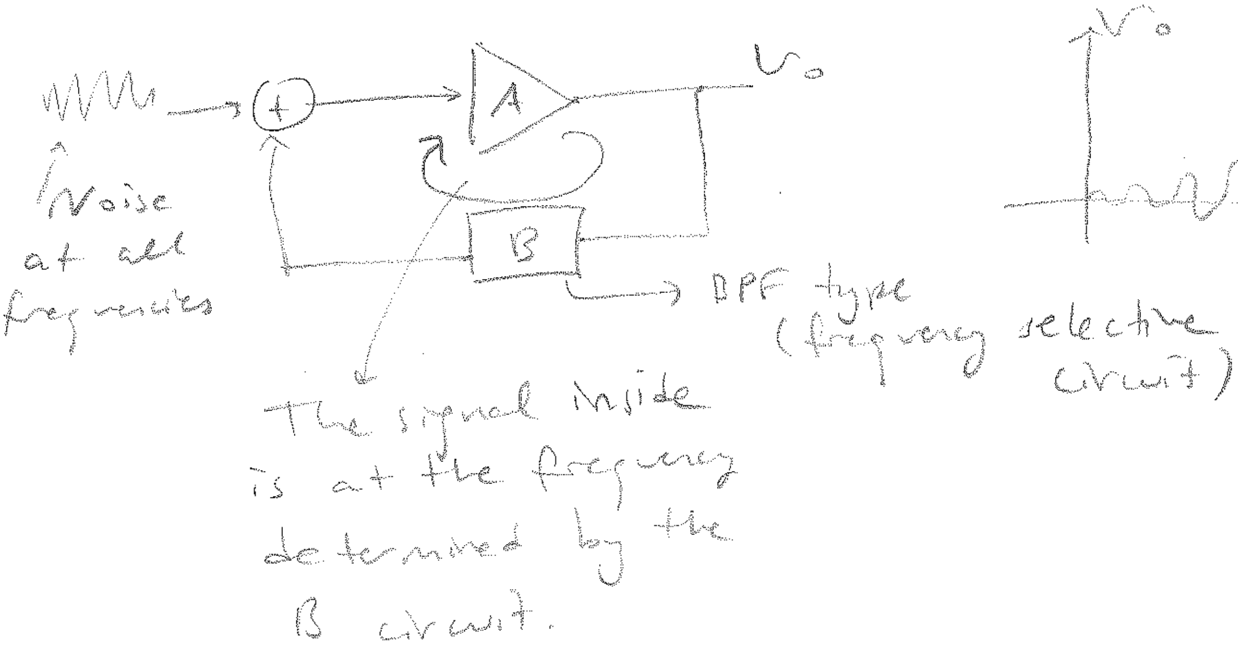
$$V_{in} + BV_o = \frac{V_o}{A}$$

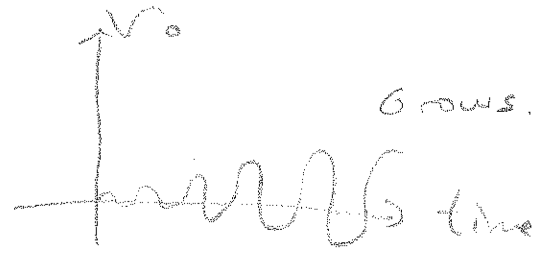
$$\rightarrow V_{in} = \frac{V_o}{A} - BV_o = V_o\left(\frac{1}{A} - B\right)$$

$$\rightarrow \frac{V_o}{V_{in}} = \frac{\cancel{V_o}}{\cancel{V_o}\left(\frac{1}{A} - B\right)} = \frac{1}{\frac{1}{A} - B} = \frac{A}{1 - AB}$$

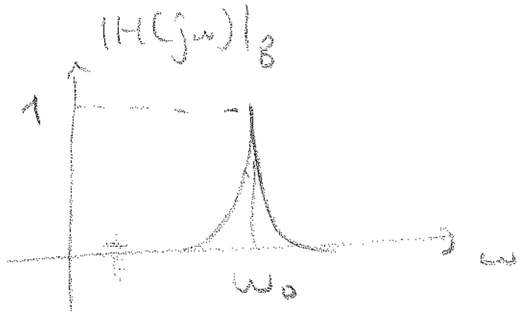
- If $V_{in} - V_f = V_i$ Then, $\frac{V_o}{V_{in}} = \frac{A}{1+AB}$ (Negative feedback Closed Loop Gain)

- If $|AB| = 1$
  $\frac{V_o}{V_{in}} = \infty$ (unstable!) which means that the signal inside the loop grows.



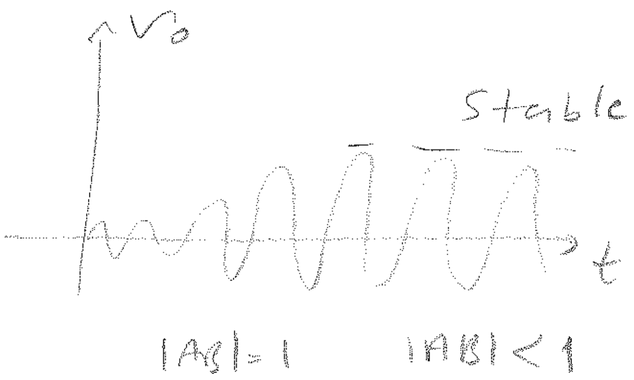


- The oscillation starts from noise (V_{in}).
- B circuit determines the frequency of oscillation.
- B circuit is a frequency selective circuit.

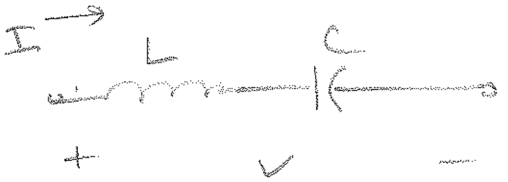


- Consider $A_v = \dfrac{A}{1 - AB}$
- For $|AB| = 1$, oscillation grows.
- As the frequency grows inside the loop the gain of the amplifier starts to decrease due to the fact that transistors enter the "SAT" region and the linearity is distorted.

As a result, $|AB| < 1$ and the gain is stabilized

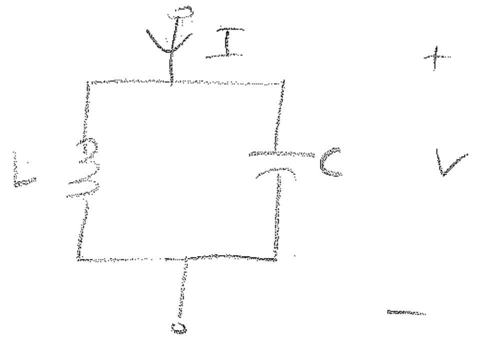


B - Circuit:



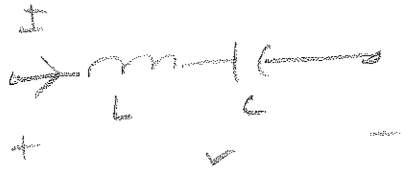
(Resonator circuit)

or



- These circuits are in fact series RLC and parallel RLC circuits.
- We have $R = 0$ for series and $R = \infty$ for parallel circuits for zero energy dissipation.

Let us consider the circuit



$$Z = \text{Impedance} = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C}$$

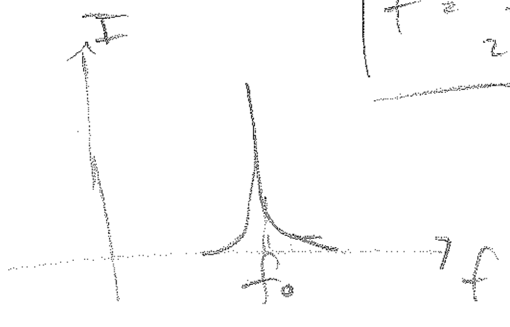
Z = 0 (Resonance):

$$1 - \omega^2 LC = 0$$

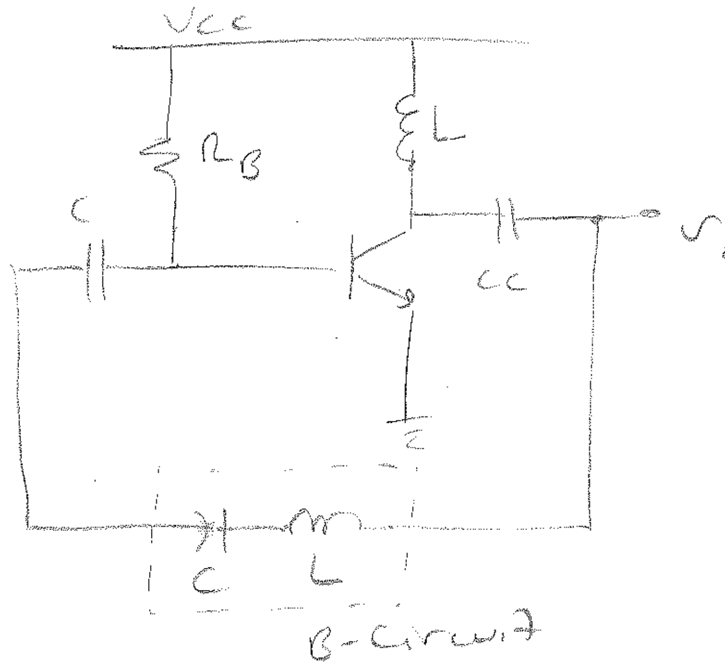
$\rightarrow \omega = \frac{1}{\sqrt{LC}}$ is the resonance frequency.

or

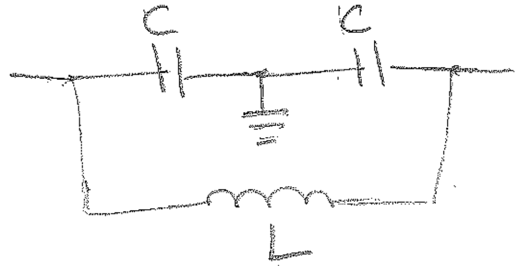
$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$ (Hz) is the Hz resonant frequency.



Implementation:



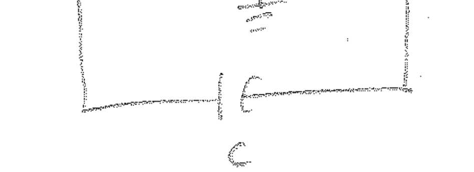
Sometimes we use:



$$f_r = \frac{1}{2\pi\sqrt{\left(\frac{C}{2}\right)L}} \ (Hz)$$

(Colpitts's Oscillator)

or we use



$$f_r = \frac{1}{2\pi\sqrt{(2L)C}} \ (Hz)$$

(Hartley's Osc.)

Ex:

In oscillator theory, the gain factor $|AB| = 1$ grows the signal. After a while $|AB| < 1$ stabilizes the signal. What causes $|AB| < 1$?

Ans: Transistors enter the "sAturation" as the signal grows.

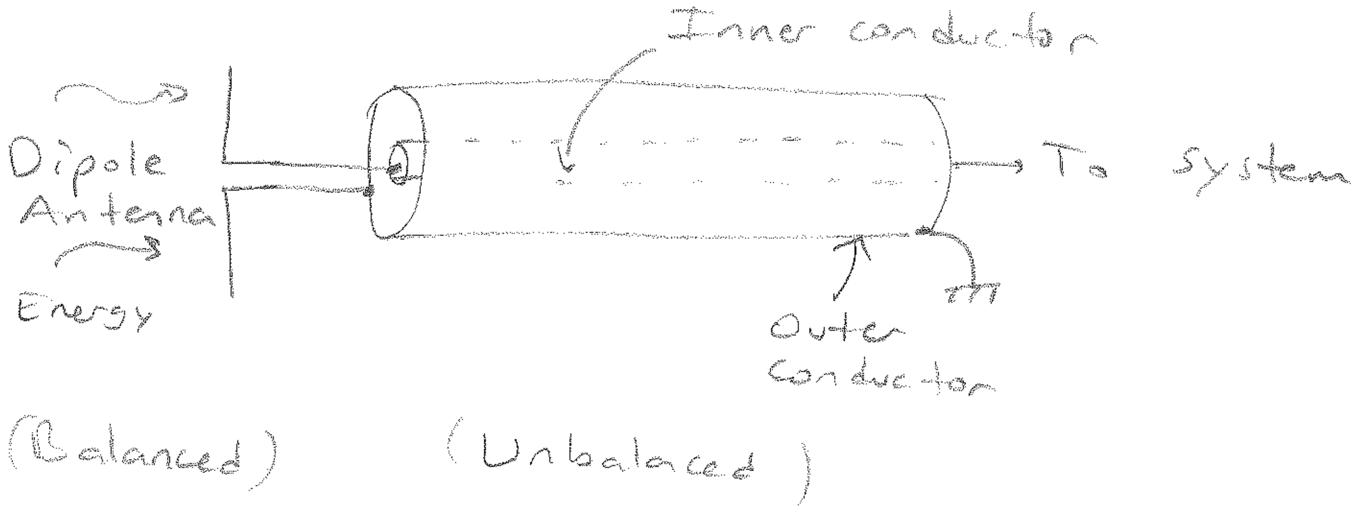
Ex:

For Colpitts' oscillator, $C = 1\mu F$ and $L = 1mH$ in the feedback circuit. Find the oscillation frequency in Hz!

Ans:

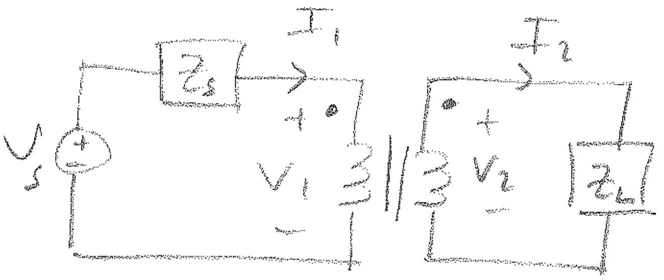
$$f_r = \frac{1}{2\pi\sqrt{\left(\frac{C}{2}\right)L}} \Rightarrow f_r = \frac{1}{2\pi\left[(0.5\times10^{-6})(1\times10^{-3})\right]^{\frac{1}{2}}} = 7.1\,kHz$$

Practical example is connecting a dipole antenna to a coaxial cable.



— Use of Transformers for Impedance Matching: —

Consider the following circuit.



$\underline{V_1} = \frac{\underline{V_2}}{a}$, $\underline{I_1} = a\underline{I_2}$

$\underline{V_1}, \underline{V_2}, \underline{I_1}, \underline{I_2}$ are phasors.

For matching:

$$\boxed{Z_{in} = \frac{1}{a} Z_L}$$